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ABSTRACT

This review guide, prepared as an aid to teachers of Course III, starts with a pre-test review of Course II topics found again in Course III. The five units of Course III, as outlined in the New York State Syllabus, are then separated into nine smaller units. These include: real numbers; complex numbers; functions; logarithms; trigonometry; geometry of the circle; transformational geometry; probability; and statistics. Each unit is separated by topic area into suggested homework assignments/activities consisting of problems drawn from past regents examinations and excerpts from available Course III, geometry, intermediate, and advanced algebra textbooks. A table of contents indicating specific topics addressed precedes each unit. Answer keys for assignments are included at the end of the guide. (JN)

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M D C² REVIEW GUIDE

COURSE III

mathematics

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A CoSer Project of the
Cortland-Madison BOCES

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MDC^2 Review Guide

Course III

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Introduction

This review guide was written to be an aid to Teachers of Course III. It starts with a Pre-test review of Course II topics found again in Course III. The five units of Course III, as outlined in the New York State Syllabus, have then been separated into smaller subtopics.

Preceeding each unit there is a Table of Contents for that particular unit. Each unit is separated into suggested homework assignments consisting of problems drawn from past regents exams and excerpts from available Course III, Geometry, Intermediate and Advanced Algebra textbooks. At the end of the guide you will find Answer Keys for these assignments.

This guide was produced through the cooperative efforts of representatives from Cincinnatus, DeRuyter and Marathon High Schools, under the auspices of Cortland-Madison BOCES Summer Curriculum Development Programs.

COURSE III PRE-TEST

NAME _____

EVALUATE:

1. $\frac{9 - 8^2}{-6 - -1}$ 1. _____

2. $(\bar{}3(7 + 5)) \div 2 + 1$ 2. _____

3. ${}_9P_2$ 3. _____

4. ${}_6C_3$ 4. _____

5. $\sqrt{20} - \sqrt{5} + \sqrt{45}$ 5. _____

6. $4!$ 6. _____

SIMPLIFY:

7. $(5r - 3) - (\bar{}2r + 4)$ 7. _____

8. $(2x - 3)^2$ 8. _____

9. $\sqrt{128x^2y}$ 9. _____

10. $\bar{}\sqrt{\frac{49}{3}}$ 10. _____

11. $(4 + \sqrt{7})(2 - \sqrt{7})$ 11. _____

12. $(\bar{}3x^2)^3$ 12. _____

SOLVE:

13. $38 + 2y = 5y - 43$

14. $3(2a + 5) + a = \bar{}13$

15. $7x \leq 5(x - 4)$

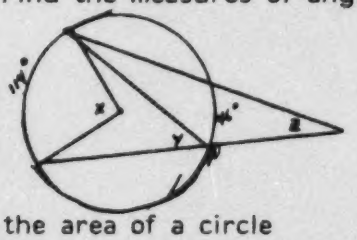
SOLVE:

16. $m^2 + 169 = 26m$

17. $c^2 + 8c + 5 = 0$

18. $\frac{n+1}{4n} + \frac{1}{n} = \frac{2}{3}$

19. - 21. Find the measures of angles x, y and z.



m ∠ x = _____
m ∠ y = _____
m ∠ z = _____

22. Find the area of a circle with diameter = 12 cm.

23. Find the circumference of a circle with radius = 7m.

24. Find the probability of getting at least a 3 when rolling a die.

25. How many 4-letter arrangements can you form form the letters of the word COMPUTER?

26. How many different permutations are there of the letters of the word GIGGLE if all letters are used?

27. How many committees of 4 students can be formed from a class of 15?

28. - 30. Find the mean, median and mode of: 37, 25, 6, 37, 10

mean = _____
median = _____
mode = _____

31. Write an equation of the circle with center $(2, -3)$ and radius = 5.

32. Graph: $y = x^2 - 4x + 1$ for the interval $-1 \leq x \leq 5$.

33. The vertices of $\triangle$ ABC are A $(-3, -1)$, B $(1,7)$ and C $(5,5)$. Without using graph paper, show that $\triangle$ ABC is a right triangle.

Course III - Pre-Test

1. 11

2. -17

3. 72

4. 12

5. 4 5

6. 24

7. $7r - 7$

8. $4x^2 - 12x + 9$

9. 8 x 24

10. $\frac{-7\ 3}{3}$

11. 1 - 2 7

12. $-27 \times {}^{6}$

13. $38 + 2y = 5y - 43$
$81 = 3y$
$27 = y$

14. $3(2a + 5) + a = -13$
$6a + 15 + a = -13$
$7a = -28$
$a = -4$

15. $7x - 5(x - 4)$
$7x - 5x - 20$
$2x - -20$
$x - 10$

16. $m^2 + 169 = 26m$
$m^2 - 26m + 169 = 0$
$(m - 13)(m - 13) = 0$
$m = 13$

17. $c^2 + 8C + 5 = 0$
$c = \frac{-8 + 64 - 20}{}$
$c = \frac{-8 + 44}{2} = \frac{-8 + 2\ 11}{2}$
$c = -4 + 11$

18. $\frac{n + 1}{4n} + \frac{1}{n} = \frac{2}{3}$

19. $3n + 3 + 12 = 8n$
$15 = 5n$
$3 = n$

19. m x = 114

20. m y = 57

21. m z = 34

22. $A = {}^{2}$
$A = 36\ cm^2$
or $113.04\ cm^2$

23. C = 2
C = 14 m
or 43.96 m

24. $\frac{2}{3}$

25. $B^P 4 = 1680$

26. $\frac{6^1}{3} = 120$

27. $15^C 4 = 1365$

28. mean = 23

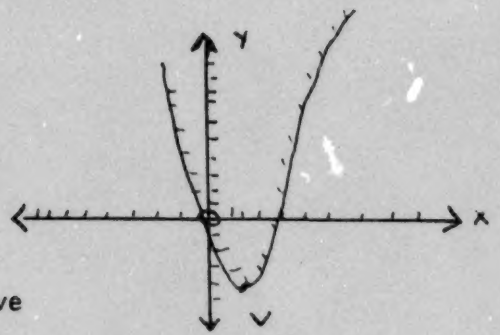
29. median = 25

30. mode = 37

31. $(x - 2)^2 + (y + 3)^2 = 25$

32. $y = x^2 - 4x + 1$
$y - 1 = x^2 - 4x$
$y - 1 = x^2 - 4x$
$y + 3 = (x - 2)^2$
$y = (x - 2)^2 - 3$

33. A (-3, -1)
B (1, 7)
C (5,5)
Slope of AB = $\frac{8}{4} = 2$
Slope of AC = $\frac{6}{8} = \frac{3}{4}$
Slope of BC = $\frac{2}{-4} = \frac{-1}{2}$
Slopes of AB and BC are negative reciprocals making B a right and ABC a right .



REAL NUMBERS
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Review of Factoring
Factor Completely

Name_____

1. $18 - 6y - 12x$
2. $x^{b+1} - x^b$
3. $100 - x^2$
4. $y^2 - 12y + 36$
5. $x^2 - 12x + 27$
6. $3y^2 + 4y + 1$
7. $2x^2 + 7x - 4$
8. $80 - 5d^2$
9. $x^3 - 121x$
10. $y^4 - 1$
11. $ax^3 - 9ax$
12. $y^{2a} - 1$
13. $x^4 - 5x^2 + 4$
14. $4x^2 + 40x + 64$
15. $2ax^2 - 8ax - 12a$
16. $9ay^2 - 21a^2y + 6a^3$
17. $x^{c+2} - 14x^{c+1} + 24x^c$
18. $x^3 + 3x^2 - 10x$
19. $2y^3 + 50y$
20. $x^{2b} - 16$
21. $y^4 - 7y^2 - 18$
22. $5x^2 - 5ax - 60a^2$
23. $8y^3 - 60y^2 - 32y$
24. $3bx^2 + b^2x^2 - 2b^3x$
25. If $25x^2 + 20d^2x + 4d^4$ represents the area of a square, express the measure of a side of the square as a binomial.

Review of Factoring
Factor Completely

NAME_____

1. $3x^2 - 12x$
2. $9y^3 + 3y^2$
3. $k^2 - 49$
4. $x^2 + 10x + 25$
5. $y^2 + 10y + 9$
6. $x^2 + 2x - 24$
7. $2k^2 - 7k + 5$
8. $3y^2 - 12$
9. $7a^2 - 7b^2$
10. $4c^3 - 36cx^2$
11. $4ab^2 - 12a^2b$
12. $y^{a+2} + 2y^a$
13. $a^2b^2 - 144$
14. $9x^2 + 6x + 1$
15. $k^2 + 5k - 14$
16. $2x^2 + 13x + 6$
17. $6y^2 - 13y - 5$
18. $100 - 4x^2$
19. $3y^3 - 192y$
20. $x^4 - 625$
21. $4x^2 - 20x + 24$
22. $36c^2 - 100d^2$
23. $x^{2+k} - 4x^k$
24. $2x^3 + 6x^2 + 2x$
25. If $3x^2 + 20x + 12$ represents the area of a rectangle, find two binomials that can represent is dimensions.

Simplifying Polnomials

Name _____

Reduce to lowest terms

1. $\frac{8x^3}{8x}$

2. $\frac{8a^2y^2}{4ay^6}$

3. $\frac{3bx^2}{(3bx)^2}$

4. $\frac{15(x - 4)}{20(x - 4)}$

5. $\frac{x - 25}{25 - x}$

6. $\frac{2y + 4}{3y + 6}$

7. $\frac{2z}{2z^2 + 6z}$

8. $\frac{16x^2}{2x^2 - 4x}$

9. $\frac{bx^3 - b^2x^2}{bx^3 - 2b^2x^2}$

10. $\frac{k^2 - 25}{k + 5}$

11. $\frac{y^2 - 81}{(y - 9)^2}$

12. $\frac{(x - 1)^2}{2 - 2x}$

13. $\frac{3a + 3}{a^2 - 1}$

14. $\frac{2p - 8}{(4 - p)^2}$

15. $\frac{(x - a)^2}{x^2 - a^2}$

16. $\frac{x^3y - x^2y^2}{xy^3 - x^2y^2}$

17. $\frac{1 - 9x^2}{3x^2 - x}$

18. $\frac{5a^2 - 45}{5a - 15}$

19. $\frac{x^2 - 81}{81 - x^2}$

20. $\frac{y^2 - 2y}{y^2 - y - 2}$

21. $\frac{2y^3 - 8y}{4y^2 - 8y}$

22. $\frac{x^2 + x - 2}{x^2 + 3x - 4}$

23. $\frac{3y^2 - 12y}{y^3 - 16y}$

24. $\frac{16 - 6x - x^2}{x^2 - 64}$

Addition and Subtraction of Polynomials Name_____

Perform the indicated operations and express in simplest form.

1. $\frac{9}{y} - \frac{3}{y}$

2. $\frac{3x}{8} + \frac{x}{4}$

3. $\frac{11b}{3y} - \frac{4b}{3y}$

4. $\frac{3}{4a} - \frac{2}{3a} + \frac{1}{6a}$

5. $\frac{1}{2x} - \frac{1}{x} + \frac{3}{8x}$

6. $\frac{x+5}{x} - \frac{8}{x^2}$

7. $\frac{c}{a} - \frac{a}{b}$

8. $\frac{x-y}{xy^2} + \frac{x+y}{x^2y}$

9. $\frac{5}{rs} + \frac{9}{st}$

10. $\frac{2y}{y-5} + \frac{10}{5-y}$

11. $\frac{x}{x+2} - \frac{8}{x^2-4}$

12. $\frac{z+3}{z-2} - \frac{10}{z^2-2z}$

13. $\frac{4}{(y^2-9)} - \frac{2}{(y^2-3y)}$

14. $\frac{1}{x^2+4x+3} + \frac{1}{x^2-1}$

15. $\frac{7}{y^2-49} - \frac{6}{y^2-2y-35}$

16. $\frac{10x}{x^2-25} + \frac{5}{5-x}$

17. $\frac{9}{(x^2+7x+10)} + \frac{3}{(x+5)} = \frac{1}{(x+2)}$

18. $\frac{2}{2y-1} - \frac{1}{2y+1} - \frac{2}{4y^2-1}$

19. $y-1 + \frac{(y-1)}{y}$

20. $2x + \frac{ax}{x-a}$

21. $y+3 - \frac{y-6}{y-2}$

22. $y-1 + \frac{1}{y+1}$

23. $\frac{2x-1}{x+2} + 2x-3$

24. $\left(4 - \frac{1}{x}\right)\left(\frac{2x}{4x-1}\right)$

25. $\left(\left(1 - \frac{5}{y}\right)\right)\left(\frac{x}{5-y}\right)$

Multiplication and Division of Polynomials Name: _____

Perform the indicated operation and express in simplest form.

1. $\frac{3b}{4b} \cdot \frac{4x}{6}$

2. $\frac{3k}{7} \div \frac{6k^2}{14}$

3. $\frac{x+5}{3a^2} \div \frac{x+5}{2a}$

4. $\frac{18a^2x}{5b^3} \cdot \frac{3b}{27ax}$

5. $\frac{2x+6}{x^2} \cdot \frac{3x^2}{6x+18}$

6. $\frac{3k-3}{K} \div \frac{12K-12}{4K^2}$

7. $\frac{(y-3)^2}{y^2-9} \div \frac{3y-9}{y+3}$

8. $\frac{xy}{x^2-4x} \cdot \frac{x^2-16}{4y}$

9. $\frac{2x^2-8x}{x^2-16} \div \frac{6x^2}{(x+4)^2}$

10. $\frac{c^3-c^2y}{c^2-y^2} \div \frac{c^3y}{cy^2+y^3}$

11. $\frac{x^2-25}{x^2+7x} \div \frac{x^2+7x+10}{x^2+9x+14}$

12. $\frac{b+8}{5b^2} \cdot \frac{3b^2-24b}{b^2-64}$

13. $\frac{x^2+1}{x^2-1} \div \frac{1}{x-1}$

14. $\frac{y^2+2y-8}{y^2+3y-4} \cdot \frac{3y^2+3y}{3y-6}$

15. $\frac{6a^2+2a}{9a^2+6a+1} \cdot \frac{9a^2-1}{6a^2}$

16. $\frac{x^3-36x}{x^2+7x+6} \div \frac{6x^2-x^3}{x^2+x}$

17. $\frac{2x-12}{3x-6} \cdot \frac{x^2-4}{x^2-36} \cdot \frac{3x+18}{4x+8}$

18. $\frac{x^2-12x+27}{x^2-81} \cdot \frac{x+9}{3-x}$

19. $\frac{x^2+4x+4}{3x^2+3x-18} \div x-2 \cdot \frac{6x+18}{x-1}$

20. $\frac{2x-8}{x^2-2x-8} \cdot \frac{x^2-4}{2y} \div \frac{x^2-2x}{xy}$

Simplifying Complex Fractions Name_____

Express in simplest:

1. $\dfrac{\frac{a^2}{b}}{\frac{a}{b^2}}$

2. $\dfrac{\frac{2}{k}}{1 + \frac{2}{k}}$

3. $\dfrac{\frac{a + b}{2a}}{\frac{a + b}{3a}}$

4. $\dfrac{\frac{24}{x - 3}}{\frac{36}{x - 3}}$

5. $\dfrac{\frac{1}{d}}{1 - \frac{1}{d}}$

6. $\dfrac{y - \frac{1}{2}}{y + \frac{1}{2}}$

7. $\dfrac{1 - \frac{2}{x}}{1 - \frac{4}{x^2}}$

8. $\dfrac{x + \frac{1}{5}}{x^2 - \frac{1}{25}}$

9. $\dfrac{\frac{1}{7} + \frac{1}{b}}{\frac{1}{b}}$

10. $\dfrac{\frac{1}{r} + \frac{1}{m}}{\frac{1}{r} - \frac{1}{m}}$

11. $\dfrac{x - \frac{1}{x}}{\frac{1 - x^2}{x}}$

16. $\dfrac{\frac{x + y}{x}}{\frac{1}{x} + \frac{1}{y}}$

17. $\dfrac{\frac{b^2}{4} - 1}{\frac{b}{4} - \frac{1}{2}}$

18. $\dfrac{\frac{2}{x^2} + \frac{2}{y^2}}{\frac{4}{xy}}$

19. $\dfrac{\frac{4}{x} - \frac{8}{x^2}}{1 - \frac{2}{x}}$

20. $\dfrac{\frac{k}{2} - \frac{k}{6}}{\frac{k}{2} + \frac{k}{3}}$

21a. $\dfrac{\frac{1}{7} - \frac{1}{x}}{\frac{x}{7} - \frac{7}{x}}$

21b. $\dfrac{1 - \frac{y}{8}}{\frac{1}{8} - \frac{1}{y}}$

22. $\dfrac{\frac{9}{2} + \frac{3}{2x}}{\frac{9x}{2} - \frac{1}{2x}}$

23. $\dfrac{\frac{a}{b} - \frac{b}{a}}{1 - \frac{b}{a}}$

24. $\dfrac{6 + \frac{12}{t}}{3t - \frac{12}{t}}$

25. $\dfrac{\frac{x}{2} - \frac{8}{x}}{\frac{1}{4} - \frac{1}{x}}$

12. $\dfrac{a^2 - b^2}{1 - \frac{b}{a}}$

13. $\dfrac{x - 5}{\frac{x}{5} - 1}$

14. $\dfrac{\frac{b}{a} - 1}{\frac{1}{a} - \frac{1}{b}}$

15. $\dfrac{\frac{y}{3} + \frac{3}{y}}{\frac{1}{3} + \frac{1}{y}}$

26. $\dfrac{\frac{3}{a^2} + \frac{5}{a^3}}{\frac{10}{a} + 6}$

27. $\dfrac{\frac{1}{r} - 3}{3n - 1}$

28. $\dfrac{1 + \frac{4}{x} + \frac{3}{x^2}}{1 - \frac{9}{x^2}}$

29. $\dfrac{1 + \frac{2}{y} - \frac{24}{y^2}}{1 + \frac{4}{y} - \frac{12}{y^2}}$

30. $\dfrac{\frac{1}{k} - \frac{3}{k^2} + \frac{2}{k^3}}{\frac{1}{k} - \frac{4}{k^2} + \frac{4}{k^3}}$

Solving Fractional Equations

Name: _____

Solve and Check:

1. $\frac{15}{y} = 3$

2. $\frac{15}{4x} = \frac{1}{8}$

3. $\frac{15}{y} - \frac{3}{y} = 4$

4. $\frac{3}{a} = \frac{19}{3a} - \frac{5}{3}$

5. $\frac{3}{2x} - 1 = \frac{x + 1}{c}$

6. $\frac{12}{x - 2} = 4$

7. $\frac{10}{2t - 1} = 2$

8. $\frac{2}{3x-4} = \frac{1}{4}$

9. $\frac{4a}{7 + 5a} = \frac{1}{3}$

10. $\frac{5}{a} = \frac{7}{a - 4}$

11. $\frac{6}{x} = 12$

12. $\frac{7}{3y} = \frac{-1}{3}$

13. $\frac{7}{c} + \frac{1}{c} = 16$

14 $\frac{y - 2}{y + 4} = \frac{y - 3}{y + 1}$

15. $\frac{8}{b} = -2$

16. $\frac{4}{5y} = \frac{-1}{10}$

17. $\frac{9}{2x} = \frac{7}{2x} + 2$

18. $\frac{4}{c} - \frac{1}{2} = \frac{5}{12} - \frac{3}{2c}$

19. $\frac{5}{x^2 - 4} - \frac{x + 12}{x + 2} + \frac{x - 1}{x - 2} = 0$

20. $\frac{x - 3}{x - 1} - \frac{x - 1}{x} = \frac{5}{x^2 - x}$

Solving Fractional Equations

Name: _____

Solve and Check:

1. $\frac{10}{x} = 5$

2. $\frac{3}{2x} = \frac{1}{2}$

3. $\frac{10}{x} + \frac{8}{x} = 9$

4. $\frac{30}{x} = 7 + \frac{18}{2x}$

5. $\frac{y + 9}{2y} + 3 = \frac{15}{y}$

6. $\frac{15}{a + 1} = 3$

7. $\frac{9}{2x + 1} = 3$

8. $\frac{6}{3x - 1} = \frac{3}{4}$

9. $\frac{3}{5 - 3a} = \frac{1}{2}$

10. $\frac{3}{y} = \frac{2}{5 - y}$

11. $\frac{b + 1}{b - 3} = \frac{b - 3}{b + 1}$

12. $\frac{2 + x}{6x} = \frac{3}{5x} + \frac{1}{30}$

13. $\frac{10}{x + 2} = 2$

14. $\frac{16}{1 - 3t} = 4$

15. $\frac{5x}{x + 1} = 4$

16. $\frac{1 - r}{1 + r} = \frac{2}{3}$

17. $\frac{2}{m} = \frac{5}{3m - 1}$

18. $\frac{x - 2}{x + 1} = \frac{x + 1}{x - 2}$

19. $\frac{1}{d - 1} = \frac{4}{d^2 - 1} - \frac{1}{d + 1}$

20. $\frac{y}{y + 1} - \frac{1}{y} = 1$

Regents - Real Numbers Name: _____

1. For which equation does the sume of the roots equal the product of the roots?

(1) $x^2 - 2x + 3 = 0$
(2) $x^2 - 13x + 13 = 0$
(3) $4x^2 - 4x + 1 = 0$
(4) $5x^2 - 5x = 0$

1. _____

2. What is the smallest integral value of x for which $\sqrt{3x - 7}$ represents a real number?

2. _____

3. Express in simplest form: $\dfrac{\dfrac{b - a}{a}}{\dfrac{1}{a} - \dfrac{1}{b}}$

3. _____

4. The solution set of $|x - 1| = 2$ is:

(1) (1, 3) (3) (-1,3)
(2) (-3,3) (4) ( )

4. _____

5. The roots of $x^2 + bx = 0$ are real when b is equal to

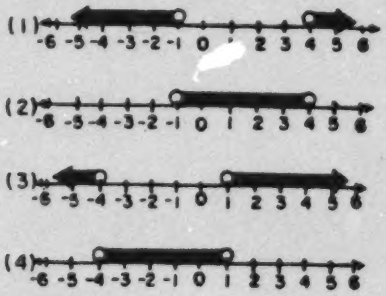
(1) 0 (3) 3
(2) 2 (4) 4

5. _____

6. Solve for x: $\sqrt{2x + 3} - 5 = 0$

6. _____

7. Which is the graph of the solution set of inequality $x^2 + 3x - 4 < 0$?



7. _____

8. Express in simplest form: $\dfrac{\dfrac{a^2 - 1}{4}}{\dfrac{a}{4} - \dfrac{1}{2}}$

8. _____

9. Which is true o the graph of $y = x^2 + 4x + 6$?
   1. It intersects the x-axis in two real points.
   2. It is tangent to the x-axis.
   3. It lies entirely below the x-axis.
   4. It lies entirely above the x-axis.

9. _____

10. The expression $\frac{a}{a - b} - \frac{b}{a + b}$ is equivalent to

(1) 1 (3) $\frac{1}{a + b}$

(2) $\frac{a + b}{a - b}$ (4) $\frac{a^2 + b^2}{a^2 - b^2}$ 10. _____

11. The equation $\sqrt{x - 2} = 2 - x$ has for its roots

(1) 2 and 3 (3) 3, only
(2) 2, only (4) neither 2 nor 3 11. _____

12. Combine and reduce to lowest terms:

$\frac{4x}{2x + 6} + \frac{x}{x + 3}$ 12. _____

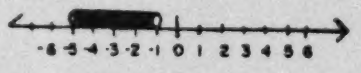
13. Express in simplest form: $\frac{\frac{a + b}{a}}{\frac{1}{a} + \frac{1}{b}}$ 13. _____

14. Solve for x: $\frac{1}{2x} = \frac{1}{x} - 2$ 14. _____

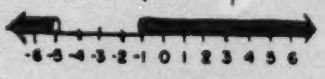
15. The graph of $y = ax^2 + bx + c$ is tangent to the x-axis. The roots of the equation $ax^2 + bx + c = 0$ are

1. real, rational, and unequal
(2) real, rational, and equal
(3) imaginary
(4) real, irrational, and unequal 15. _____

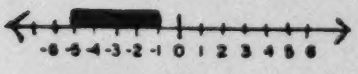
16. Which is the graph of the solution set of $|x + 3| > 2$? 16. _____



(1)



(3)



(2)

(4)

17. Solve for x: $\frac{1}{15} + \frac{1}{10} = \frac{1}{x}$ 17. _____

18. Express in simplest form: $\dfrac{\frac{1}{2} + \frac{1}{x}}{\frac{1}{x}}$

18. _____

19. Express in simplest form: $\dfrac{1 - \frac{2}{x}}{\frac{1}{x}}$

19. _____

20. Express in simplest form: $\dfrac{\frac{1}{x^2} - 1}{1 + \frac{1}{x}}$

20. _____

Regents - Real Numbers

Name: _____

1. Find the solution set for $|2x - 1| = 3$. 1. _____

2. Solv for x: $\frac{1}{x} + 1 = \frac{3}{2x}$ 2. _____

3. Express in simplest form: $\dfrac{n - \frac{1}{n^2}}{\frac{1 - n^2}{n}}$ 3. _____

4. Solve for x: $\sqrt{6x - 2} = 4$ 4. _____

5. The roots of $x^2 + bx + 2 = 0$ will be imaginary when b equals

(1) 1 (3) 3
(2) 7 (4) 9 5. _____

6. Express in simplest form: $\dfrac{3 - \frac{3}{x}}{x - 1}$ 6. _____

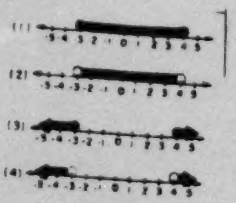
7. Perform the indicated operations and reduce completely:

$$\frac{x^2 + 4x = 3}{x^2 + x} \cdot \frac{12 - 3x}{2x} \div \frac{x^2 - x - 12}{4x^2}$$ 7. _____

8. If $a > b$ and $c < 0$, then which statement is true?

(1) $a + c < b + c$ (3) $ac > bc$
(3) $a - c < b - c$ (4) $ac < bc$ 8. _____

9. Which is the graph of the solution set of $|1 - 2x| < 7$? 9. _____



10. What is the smallest integral value of x for which $\sqrt{2x - 3}$ represent a real number?

10. _____

11. The roots of $x^2 + x + 1 = 0$ are

(1) real, rational, and unequal
(2) real, irrational, and unequal
(3) real, rational, and equal
(4) imaginary

11. _____

12. If the graph of $y = 2x^2 + 4x + c$ does <u>not</u> intersect the x-axis, then c may be equal to

(1) 1 (3) -2
(2) 2 (4) 5

12. _____

13. Express in <u>simplest form</u>: $\dfrac{\dfrac{a - b}{a}}{\dfrac{a - 1}{b}}$

13. _____

14. If the roots of the equation $ax^2 + bx + c = 0$ are imaginary, the graph of the equation will

(1) intersect the x-axis at two points
(2) not intersect the y-axis
(3) not intersect the x-axis
(4) be tangent to the x-axis

14. _____

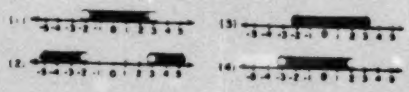
15. Express in <u>simplest form</u>: $\dfrac{\dfrac{1}{x}}{1 + \dfrac{1}{x}}$

15. _____

16. Express in simplest form: $\dfrac{\dfrac{x}{4} - \dfrac{1}{x}}{1 - \dfrac{2}{x}}$

16. _____

17. Which is the graph of the solution set of $x^2 - x - 6 < 0$?



17. _____

18. The equation $x = \sqrt{2x + 3}$ has

(1) 3 and -1 as solutions
(2) -1 as its only solution
(3) 3 as its only solution
(4) no solution

18. _____

19. Which represents the solution set for x in the inequality $|2x - 1| < 7$?

(1) $([x]x < -3$ or $x > 4)$
(2) $([x]x < -4$ or $x > 3)$
(2) $([x] - 4 < x < 3)$
(4) $([x[- 3 < x < 4)$

19. _____

COMPLEX NUMBERS
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Simplifying Radicals

Name: _____

Simplify the Expression:

1. $\sqrt{128}$

2. $-\sqrt{500}$

3. $5\sqrt{24}$

4. $-6\sqrt{98}$

5. $\frac{1}{2}\sqrt{18}$

6. $\frac{-2}{5}\sqrt{125}$

7. $3\sqrt{mn^2}$

8. $-4\sqrt{2a^2b^3}$

9. $5\sqrt{4x^2y^3}$

10. $-7\sqrt{24a^2b^3}$

11. $\sqrt{\frac{12}{5}}$

12. $-\sqrt{\frac{7}{12}}$

13. $\sqrt{\frac{8}{7}}$

14. $-\sqrt{\frac{5}{18}}$

15. $4\sqrt{\frac{25}{2}}$

16. $-\sqrt{\frac{3}{32}}$

17. $12\sqrt{\frac{c}{6}}$

18. $-15\sqrt{\frac{16}{5}}$

19. $10\sqrt{\frac{25}{20}}$

20. $-8\sqrt{\frac{x^2}{2}}$

Addition/Subtraction Radicals

Name: _____

Combine the Radicals:

1. $5\sqrt{3} + \sqrt{3} - 2\sqrt{3}$
2. $3\sqrt{5} + 6\sqrt{2} - 3\sqrt{2} + \sqrt{5}$
3. $15\sqrt{y} - 7\sqrt{y}$
4. $\sqrt{2} + \sqrt{50}$
5. $\sqrt{5} + \sqrt{45} + \sqrt{80}$
6. $\sqrt{72} - \sqrt{50}$
7. $3\sqrt{2} + 2\sqrt{32}$
8. $5\sqrt{27} - \sqrt{108} + 2\sqrt{75}$
9. $3\sqrt{8} - \sqrt{2}$
10. $3\sqrt{50} - 5\sqrt{18}$
11. $4\sqrt{7} - \sqrt{7} - 5\sqrt{7}$
12. $9\sqrt{x} + 3\sqrt{x}$
13. $7\sqrt{x} + 3\sqrt{y} - \sqrt{x} - 3\sqrt{y}$
14. $\sqrt{27} + \sqrt{75}$
15. $\sqrt{80} - \sqrt{5}$
16. $\sqrt{12} - \sqrt{48} + \sqrt{3}$
17. $3\sqrt{32} - 6\sqrt{8}$
18. $3\sqrt{40} - \sqrt{90}$
19. $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$
20. $3\sqrt{28} - 2\sqrt{63}$

Multiply/Divide Radicals

Name: _____

Perform the indicated operation:

1. $\sqrt{21} \cdot \sqrt{3}$

2. $5\sqrt{8} \cdot 7\sqrt{3}$

3. $5\sqrt{6} \cdot \frac{2}{3}\sqrt{15}$

4. $7\sqrt{56} \cdot 3\sqrt{\frac{1}{2}}$

5. $\sqrt{60} \cdot \sqrt{5}$

6. $3\sqrt{3} \cdot \sqrt{18}$

7. $\frac{1}{3}\sqrt{18} \cdot 12\sqrt{3}$

8. $3\sqrt{6} \cdot \sqrt{3}$

9. $\frac{2}{3}\sqrt{24} \cdot 9\sqrt{3}$

10. $6\sqrt{\frac{1}{3}} \cdot 4\sqrt{12}$

11. $\frac{12}{2\sqrt{3}}$

12. $\frac{18}{3\sqrt{2}}$

13. $\frac{36}{\sqrt{18}}$

14. $\frac{60}{3\sqrt{8}}$

15. $\frac{25}{\sqrt{20}}$

16. $\frac{\sqrt{6}}{\sqrt{5}}$

17. $\frac{30}{\sqrt{50}}$

18. $\frac{15\sqrt{3}}{\sqrt{5}}$

19. $\frac{\sqrt{5} - \sqrt{7}}{\sqrt{5}}$

20. $\frac{4\sqrt{3} + 2}{\sqrt{3}}$

Solving Radical Equations

Name: _____

Solve and Check:

1. $\sqrt{2x} = 8$
2. $4\sqrt{3x} = -9$
3. $\sqrt{r + 3} = 5$
4. $\sqrt{2b - 4} = 10$
5. $\sqrt{2r} + + 7 = 8$
6. $2\sqrt{5x} = 20$
7. $\sqrt{b - 1} = 8$
8. $\sqrt{3a + 3} = 6$
9. $\sqrt{m} - 6 = 2$
10. $\sqrt{x + 1} = 4$
11. $\sqrt{2x - 1} = 7$
12. $\sqrt{z} + 3 = 8$
13. $2\sqrt{z} + 1 = 5$
14. $\sqrt{3x - 8} = \sqrt{x}$
15. $\sqrt{5y} = \sqrt{2y + 6}$
16. $\sqrt{2x + 1} - 1 = 4$

Addition/Subtraction Complex Numbers Name: _____

In 1-14, perform the operation and express the result in the form a + bi:

1. $(10 + 3i) + (5 + 8i)$

2. $(7 - 2i) + (3 - 6i)$

3. $(4 - 2i) + (-3 + 2i)$

4. $(6 + 3i) - (2 + i)$

5. $(-8 + 5i) - (5 - 7i)$

6. $(9 - 2i) - (9 - 5i)$

7. $(1.3 + 4i) + (2.9 - 1.7i)$

8. $(3.1 - 0.6i) - (4.8 - 0.4i)$

9. $\left(\frac{2}{3} - \frac{i}{4}\right) + \left(\frac{1}{6} - \frac{i}{2}\right)$

10. $\left(\frac{4}{5} + \frac{3}{8}i\right) - \left(\frac{3}{10} + \frac{3}{4}i\right)$

11. $(8 + \sqrt{-9}) - (10 + \sqrt{-4})$

12. $(-2 + \sqrt{-12}) + (8 + \sqrt{-27})$

13. $(-1 - \sqrt{-80}) - (3 + \sqrt{-20})$

14. $(5 - \sqrt{-128}) + (-5 - \sqrt{-98})$

15. Add: 5 + i, 7 - 3i, 12 + 6i, -10 - 8i, and -14 + 4i.

16. Subtract 2 - 13i from 7 - 5i.

17. From the sum of 3 - i and -2 - 2i, subtract 4 - 5i.

18. Express the sum of $9 + \sqrt{-9}$ and $5 - \sqrt{-16}$ in the from a + bi.

19. Express the difference, $(5 - \sqrt{-50} - (-2 + \sqrt{-162})$, in the form a + bi.

Multiplication Complex Numbers Name:_____

In 1-13, write the product of the given numbers in the form a + bi:

1. $(3 + i)(4 + i)$
2. $(5 - i)(3 + 1)$
3. $(1 + 3i)(2 - i)$
4. $(4 - 5i)(2 + i)$
5. $(7 - i)(1 - 2i)$
6. $(8 - 5i)(3 - i)$
8. $(6 - i)(6 + i)$
9. $(2 - i)(4 + 2i)$
10. $(8 + \sqrt{-25})(2 + \sqrt{-1})$
11. $(3 - \sqrt{-49})(2 + \sqrt{-16})$
12. $(2 + \sqrt{-9})(3 - \sqrt{-4})$
13. $(5 - \sqrt{-36})(2 - \sqrt{-100})$

Perform the indicated operations and express in a + bi form or as a real number whenever possible:

14. $(3 - 2i)^2$
15. $(2 - 2i)^2$
16. $(1 + 6i)(6 + i)$
17. $(4 - 3i)(3 - 4i)$
18. $(2 + 0i)(8 - 5i)$
19. $i(1 + i) + i^3$
20. $8 + i(8 - i)$

Division of Complex Numbers Name: _____

In 1-8, write the multiplicative inverse of the given complex number in the form a + bi. Simplify, if possible:

1. $3 + i$

2. $6 - i$

3. $1 + 5i$

4. $4 - 3i$

5. $9 - 2i$

6. $7 + 5i$

7. $4 - 4i$

8. $5 + 5i$

Perform the given division:

9. $\frac{1 - 3i}{2 - 7i}$

10. $\frac{3 - 5i}{i}$

11. $\frac{5 + i}{5 - i}$

12. $\frac{7 + 2i}{4i}$

13. $\frac{5 - 3i}{1 - i}$

14. $\frac{18 + i}{2 + 3i}$

15. $\frac{4 + i}{2 - 3i}$

16. $\frac{7 - i}{7 + i}$

17. $\frac{6 + i}{6 - i}$

18. $\frac{1 + 3i}{2 + 4i}$

19. $\frac{21 + i}{5 - i}$

20. $\frac{7 - 4i}{1 - 2i}$

Roots of Quadratic Equations Name:_____

In 1-8, select the numberal preceding the expression that best completes the sentence:

1. The roots of $x^2 = 2x + k = 0$ are equal when it is:
   (1) 1 (20 2 (3) 3 (4) 4

2. The roots of $x^2 + kx + 3 = 0$ are real when it is:
   (1) 1 (2) 2 (3) 3 (4) 4

3. The roots of $x^2 + bx + 8 = 0$ are imaginary when b equals:
   (1) 8 (2) -7 (3) 3 (4) -1

4. The roots of $ax^2 + 6x + 4 = 0$ are imaginary if a equals:
   (1) 1 (2) 2 (3) 3 (4) -1

5. If the graph of $y = ax^2 + bx + c$ is tangent to the x-axis, then the roots of $ax^2 + bx + c = 0$ are:
   (1) rational and unequal (2) rational and equal
   (3) irrational and unequal (4) imaginary

6. The graph of $y = ax^2 + bx + c$ where $a = 0$ lies entirely above the x-axis. The roots of $ax^2 + bx + c = 0$ are:
   (1) rational and unequal (2) rational and equal
   (3) irrational (4) imaginary

7. The graph of $y = x^2$ is symmetric with respect to:
   (1) the x-axis (2) the y-axis
   (3) the line $y = x$ (4) the point (0,0)

8. If the roots of $x^2 + bx + 16 = 0$ are equal, the b is:
   (1) 8, only (2) -8, only (3) 8 or -8 (4) neither 8 nor -8

In 9-20, wirte a quadratic equation whose roots are given below:

9. -8, 8

10. 7,0

11. $\sqrt{3}, -\sqrt{3}$

12. $\frac{1}{2}, 4$

13. $-2, \frac{3}{2}$

14. $\frac{-1}{3}, -\frac{2}{3}$

15. i, -i

16. 4i, -4i

17. 2 + i, 2 - i

18. 6 - i, 6 + 1

19. $2 + \sqrt{3}, 2 - \sqrt{3}$

20. $1 - \sqrt{7}, 1 + \sqrt{7}$

Discriminant and Roots Name: _____

Find the value of the discriminant of each equation and state the number and nature of the roots:

1. $x^2 + 3x - 5 = 0$
2. $x^2 - 10x + 25 = 0$
3. $x^2 + 3x + 4 = 0$
4. $x^2 - 10x + 25 = 0$
5. $x^2 + 8x + 17 = 0$
6. $x^2 + 7x - 30 = 0$
7. $x^2 = 7x - 6$
8. $2x^2 + 3x = 4$
9. $4x^2 + 9 = 12x$
10. $x^2 + 9 = 2x^2 + x$

Solve and Check:

11. $x^2 - 6x + 13 = 0$
12. $x^2 = 2x + 6$
13. $9x^2 + 4 = 0$
14. $x^2 + 4x = x$
15. $x^2 + 20x = 6x - 49$

Solving Quadratic Equations

Name: _____

Solve and Check:

1. $x^2 - 6x + 10 = 0$

2. $2x^2 = 6x - 5$

3. $x^2 + 10x + 26 = 0$

4. $\frac{16x}{7} + \frac{7}{x} = 0$

5. $x^2 + 29 = 4x$

6. $\frac{x^2}{4} = 3x - 10$

Regents - Complex Numbers Name: _____

Multiple Choice: Write the number of the correct answer:

1. When the $\sqrt{-3}$ is subtracted from $\sqrt{-12}$, the difference is: 1. _____

(1) $i\sqrt{3}$ (3) $3i\sqrt{3}$
(2) $-1\sqrt{3}$ (4) $-3i\sqrt{3}$

2. The expression $\sqrt{-128}$ is equivalent to 2. _____

(1) $2\sqrt{8}$ (3) $8i\sqrt{2}$
(2) $-8\sqrt{2}$ (4) $-8\sqrt{2}$

3. The expression $\frac{5}{2 - \sqrt{3}}$ is equivalent to 3. _____

(1) $10 + 5\sqrt{3}$ (3) $-10 - 5\sqrt{3}$
(2) $-2 - \sqrt{3}$ (4) $2 + \sqrt{3}$

4. The expression $\frac{3}{2 - \sqrt{3}}$ is equivalent to 4. _____

(1) $\frac{6 + 3\sqrt{3}}{7}$ (3) $6 + 3\sqrt{3}$
(2) $\frac{6 - 3\sqrt{3}}{7}$ (4) $6 - 3\sqrt{3}$

5. The expression $\frac{5}{2 - \sqrt{3}}$ is equivalent to 5. _____

(1) $10 + 5\sqrt{3}$ (3) $-10 - 5\sqrt{3}$
(2) $-2 - \sqrt{3}$ (4) $2 + \sqrt{3}$

6. What is the multiplicative inverse of $2 + i$? 6. _____

(1) $2 - i$ (2) $\frac{2 - i}{3}$
(2) $-2 - i$ (4) $\frac{2 - i}{5}$

7. The expression $\frac{7}{2 - \sqrt{2}}$ is equivalent to 7. _____

(1) $7 - 7\sqrt{2}$ (3) $\frac{14 + 7\sqrt{2}}{2}$
(2) $\frac{14 - 7\sqrt{2}}{2}$ (4) $7 + 7\sqrt{2}$

8. The expression $\frac{5}{4 - 3i}$ is equivalent to 8. _____

9. Express in a + bi form: $(3 - 2i)(1 + i)$ 9. _____

10. Express the product of $(2 + \sqrt{-9})$ and $(3 - \sqrt{-16})$ in the form a + bi: 10. _____

11. Express $(3 - 2i)^2$ in the form a + bi) 11. _____

12. What is the value of c if $x^2 - 4x + c = 0$ and the roots of the equation are (2 + i) and (2 - i)? 12. _____

13. Express in terms of i the sum of $\sqrt{-16} + 3\sqrt{-25}$. 13. _____

14. Express the product of (4 - 5i) and (2 + i) in the form a + bi. 14. _____

SHOW ALL WORK:

15. Write, in the form $ax^2 + bx + c = 0$, a quadratic equation whose roots are 3 + i and 3 - i. 15. _____

16. Solve the equation $x^2 - 2x + 5 = 0$ and express the roots in the form a + bi. 16. _____

17. Solve the equation $\frac{x^2}{2} = x - 1$ and express its roots in the form a + bi. 17. _____

18. Solve the equation $2x^2 - 6x + 5 = 0$ and express its roots in terms of i. 18. _____

Regents - Complex Numbers Name: _____

1. Express as a fraction with a rational denominator $\frac{5}{3 - \sqrt{3}}$ 1. _____

2. Express the product of $(2 + i)$ and $(1 - i)$ in an $a + bi$ form. 2. _____

3. Express in terms of i the sum of $\sqrt{-9} + 2\sqrt{-64}$. 3. _____

4. Express in simplest form: $\frac{9}{\sqrt{27}} + \sqrt{12}$: 4. _____

5. Express the product of $(2 + \sqrt{-9})$ and $(3 - \sqrt{-16})$ in the form $a + bi$. 5. _____

6. Express the sum of $(3 + \sqrt{-16})$ and $(7 - \sqrt{-81})$ in the form $a + bi$. 6. _____

7. If 1 and i and 1 - i are the roots of the quadratic equation $x^2 + bx + c = 0$, what is the value of c? 7. _____

8. The square of $(2 - 2i)$ is 8. _____

9. The sum of $\sqrt{-2}$ and $\sqrt{-18}$ is 9. _____

10. The product of $(2 - 2i)$ and $(2 + 2i)$ is 10. _____

11. The complex number $i^3 + i(2 - i)$ is equivalent to 11. _____

(1) $1 + i$ (3) $-1 + 3i$
(2) $1 + i$ (4) $1 + 3i$

12. The expression $\frac{2}{3 + \sqrt{2}}$ is equivalent to 12. _____

(1) $6 + 2\sqrt{2}$ (3) $\frac{6 + 2\sqrt{2}}{7}$
(2) $6 - 2\sqrt{2}$ (4) $\frac{6 - 2\sqrt{2}}{7}$

13. The expression $\frac{2}{3 - \sqrt{3}}$ is equivalent to 13. _____

(1) $3 + \sqrt{3}$ (3) $\frac{3 + \sqrt{3}}{3}$
(2) $\frac{3 + \sqrt{3}}{2}$ (4) $\frac{1}{3}$

14. The expression $\frac{1}{4 + i}$ is equivalent to 14. _____

(1) $4 - i$ (3) $\frac{4 + i}{17}$
(2) $\frac{4 - i}{15}$ (4) $\frac{4 - i}{17}$

Show <u>all</u> work:

15. Solve the equation $x^2 + 8x + 20 = 0$ and express its roots in the form $a + bi$. 15. _____

16. Write a quadratic equation whose roots are $2i$ and $-2i$. 16. _____

17. Wolve the equation $3x^2 + 3 = 5x$ and express its roots in terms of i. 17. _____

18. Find in $a + bi$ form the roots of the equation $x^2 - 2x + 10 = 0$. 18. _____

FUNCTIONS
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Functions: Types, Evaluation, Range, Domain Name:_____

In 1-9: a. Identify the function as being a linear, constant, quadratic, absolute-value, or step function. b. Find f(3), c, Find f ($\frac{1}{2}$).

1. $f(x) = 2$ 1.a._____ b._____ c._____

2. $f(x) = x$ 2.a._____ b._____ c._____

3. $f(x) = [x]$ 3.a._____ b._____ c._____

4. $f(x) = 2x^2$ 4.a._____ b._____ c._____

5. $f(x) = 2 - x$ 5.a._____ b._____ c._____

6. $f(x) = |2 - x|$ 6.a._____ b._____ c._____

7. $f(x) = [2 - x]$ 7.a._____ b._____ c._____

8. $f(x) = x^2 - x$ 8.a._____ b._____ c._____

9. $f(x) = \pi$ 9.a._____ b._____ c._____

In 10-12, for the given function and its domain, find the range.

10. $\{(x,y) \mid y = 3x - 2\}$; domain = $(x) -2 \leq x \leq 2)$ 10. _____

11. $f(x) = 5 - \frac{1}{2}x$; domain = $\{x \mid 0 \leq x \leq 16)$ 11. _____

12. $g: x \rightarrow 6 - x$; domain = $\{x \mid x \geq 0)$ 12. _____

13. If the domain of $y = 4x - 3$ is $\{x \mid 2 \leq x \leq 5\}$, what is the greatest value in its range? 13. _____

14. The domain of $f(x) = 9 - 2x$ is $4 \leq x \leq 10$. What is the greatest value in the range of f? 14. _____

15. Let the domain of the quadratic function $y = 1 + 4x - x^2$ be $0 \leq x \leq 4$. a. Graph the function, including all points whose x-coordinates are 0, 1, 2, 3, 4. b. What is the greatest value in the range of this function? 15.a. _____ b. _____

In 16-18, state the largest possible domain such that the given relation is a function.

16. $f(x) = \frac{2}{x - 2}$ 16. _____

17. $g(x) = \frac{x - 3}{x - 9}$ 17. _____

18. $b(x) = \frac{x}{x + 7}$ 18. _____

In 19-24, for the given function: a. State the domain. b. State the range.

19. $y = 2x$ 19.a. _____ b. _____

20. $y = x^2$ 20.a. _____ b. _____

21. $y = \sqrt{x}$ 21.a. _____ b. _____

22. $h(x) = |x - 5|$ 22.a. _____ b. _____

23. $f(x) = 10$ 23.a. _____ b. _____

24. $x \xrightarrow{g} x + 10$ 24.a. _____ b. _____

Applications Name: _____

1. A newspaper deliverer earns $.07 for each paper delivered. Thus, the earnings E is a function of the number n of newspapers delivered, or E = f(n). The formula to determine the deliverer's earnings is E = .07n, or f(n) = .07n.
   a. How much is earnined when 20 papers are delivered? That is, find f(20).
   b. Find F(15). c. Find f(32). d. Find f(57).
   e. If Jennifer earns $2.66 for a daily delivery, how many newspapers does she deliver each day?

1.a. _____
b. _____
c. _____
d. _____
e. _____

Amount of Sale	8% Sales Tax
$.01 to $.10	NONE
.11 to .17	$.01
.18 to .29	.02
.30 to .42	.03
.43 to .54	.04
.55 to .67	.05
.68 to .79	.06
.80 to .92	.07
.93 to 1.06	.08

2. The accompanying chart shows an 8% sales tax to be collected on amounts from $.01 to $1.06. On sales over $1.06, the tax is computed by multiplying the amount of sale by .08 and rounding to the nearest whole cent.
   The sales tax t is a function of the amount A of te sale, that is, t = f(A).
   a. Find the sales tax on an item costing $.52, that is, find f($.52).
   b. Find f($.89).
   c. Find f($.39).
   d. Find f($.08).
   e. Find f($9.95).
   f. True or False: Sales tax is an example of a step function.
   g. True or False: For amounts less than $1.00, the tax in the chart is equal to 8% of the amount, rounded to the nearest whole cent.

2.a. _____
b. _____
c. _____
d. _____
e. _____
f. _____
g. _____

3. The cost of sending a telegram is $7.45 for 15 words or less, and $.25 for each additional word. Thus, the cost c is a function of the number of words, w, or c = f(w).
   a. Find the cost to send a telegram of 17 words that is, find f(17).
   b. Find f(21).
   c. Find f(13).
   d. Find f(38).
   e. If a telegram costs $10.45 how many words does it contain?

3.a. _____
b. _____
c. _____
d. _____
e. _____

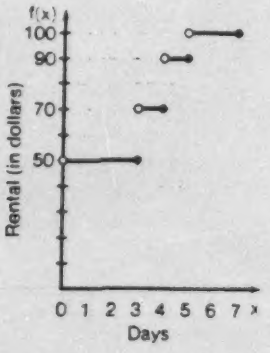
4. A mailgram costs $3.90 for 50 words or less, and $1.25 for each additional 50 words or part thereof. The cost c is a function of the number of words w, or c = f(w).
   a. Find the cost of a milgram containing 38 words, that is, find f(38).

4.a. _____

4. (Continued)
   b. Find f(65). b._____
   c. Find f(170). c._____
   d. Find F(114). d._____
   e. What is the amximum number of words in a mailgram costing $8.90? e._____
   f. What is the minimum number of words in a mailgram costing $8.90? f._____

5. If an object is dropped from a height, the distance d that it travels is a function of the time t for its fall, or d = f(t). In Earth's gravity, this distance is found by the formula $d = 16t^2$, where d is distance in feet and t is time in seconds. Assume tha an object is dropped from the top of a tall building.
   a. How many feet will the object travel in 1 second? Or, find f(1). 5.a._____
   b. Find f(2). b._____
   c. Find f(3). c._____
   d. Find f(4). d._____
   e. If the object hits the ground in 7 seconds how tall is the building? e._____

6. A car rental agency charges $50 for a rental of 3 days or less and $20 for each additional day or part there of. A weekly rental costs $100. The graph at the right represents the charges for rentals of one week or less.
   a. True or False: The rental fee r is a function of the number of days d of rental, or r = f(d). 6.a._____
   b. Find the rental fee for 4 days, that is, find f(4). b._____
   c. Explain why f(6) = $110. c._____
   d. State the range of the function. d._____



7. The rates charged to park a car in a city lot are shown in the accompanying sign. The rate r is a function of the number of house h that the car was parked, or r = f(h).
   a. Graph the function for a 24-hour period.
   b. Find the rate to park a car for $2\frac{1}{2}$ hours, that is, find $f(2\frac{1}{2})$.
   c. Find $f(4\frac{1}{4})$.
   d. Find f(7).
   e. Find $f(10\frac{1}{2})$.

Hours	Rate
First 2 hours or less	$3.00
Each additional 1 hour or part	$1.00
Maximum for 24 hours	$7.00

7.a. _____
b. _____
c. _____
d. _____
e. _____

8. A local telephone call from a home phone, under timed service, costs $.08 for the first 5 minutes or less and $.01 for each additional minute or any part thereof.
   a. Graph this step function for costs of telephone calls lasting 12 minutes or less.
   b. True or False. The cost is a function of the time of the call.
   c. Using these rates, find the call of a call lasting:
      (1) $6\frac{1}{2}$ minutes
      (2) 10 minutes 45 seconds
      (3) 1 hour

8.a. _____
b. _____
c.1. _____
2. _____
3. _____

Inverse functions

Name: _____

In 1-4, write the inverse of the given function.

1. $\{(1-5), (2,7), (3, -2), (4, -3)\}$ 1. _____

2. $\{(0,6), (4,2), (-1,7), (-2,8)\}$ 2. _____

3. $\{(1,1), (2,2), (3,3), (4,4)\}$ 3. _____

4. $\{(1,k), (2,k + 1), (3,k + 2)\}$ 4. _____

5. Let $f = \{(3, -2), (4, -2), (5, -1)\}$.
   a. Write the relation formbed by interchanging the x-coordinate and y-coordiante of f. 5.a. _____
   b. Is this new relation a function? If not, explain why. b. _____

In 6-14 write the equation of the inverse of the given function, solved for y.

6. $y = 3x$ 6. _____

7. $y = 1/3x + 1$ 7. _____

8. $y = \sqrt{x}$ 8. _____

9. $y = x - 6$ 9. _____

10. $y = 4x - 8$ 10. _____

11. $y - 12 = 3x$ 11. _____

12. $y = \frac{1}{4}x$ 12. _____

13. $y = 5 - x$ 13. _____

14. $y + 7 = x^3$ 14. _____

15. If $f(x) = 3x - 7$, evaluate:
    a. $f(2)$
    b. $f^{-1}(-1)$

15.a. _____
b. _____

16. If $g(x) = 2/3x + 4$, evaluate:
    a. $g(-3)$
    b. $g^{-1}(2)$

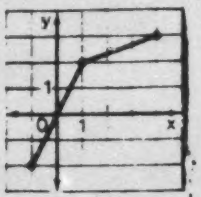
16.a. _____
b. _____

17. If $h(x) = 5x - 2$, find the value of $(h^{-1} \circ h)(123)$.

17. _____

In 18-20: a. On graph paper copy the function f. b. Using the same axes, sketch the graph of f^{-1}, the inverse under composition. c. State the domain and range of f. d. State the domain and range of f^{-1}.

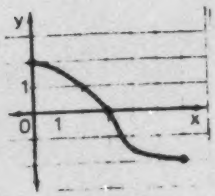
18.



18.c. _____
d. _____

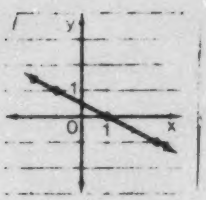
19.c. _____
d. _____

19.



20.c. _____
d. _____

20.



Function Composition Name: _____

In 1-10, for the given functions ((x) and g(x), find the rule of the composition (f g)(x).

1. $f(x) = 6x$; $g(x) = x - 2$ 1. _____

2. $f(x) = x - 10$; $g(x) = 4x$ 2. _____

3. $f(x) = x$; $g(x) = 2x + 5$ 3. _____

4. $f(x) = x - 3$; $g(x) = x - 5$ 4. _____

5. $f(x) = 2x$; $g(x) = 5x$ 5. _____

6. $f(x) = 3x + 2$; $g(x) = x - 3$ 6. _____

7. $f(x) = \frac{1}{2}x - 3$; $g(x) = 4x + 6$ 7. _____

8. $f(x) = 5 - x$; $g(x) = x + 2$ 8. _____

9. $f(x) = x^2$; $g(x) = x - 5$ 9. _____

10. $f(x) = 4 - x^2$; $g(x) = x - 2$ 10. _____

11. If $f(x) = x + 8$, then the rule of the composition $(f \circ f)(x)$ is: 11. _____
(1) $x + 8$ (2) $x + 16$ (3) $2x + 8$ (4) $2x + 16$

12. If $g(x) = 2x + 5$, what is the rule of the composition $(g \circ g)(x)$? 12. _____

13. Let $h(x) = x^2 + 2x$, and $g(x) = x - 3$. In a to e evaluate the composition.
a. $(h \circ g)(4)$ 13.a. _____
b. $(h \circ g)(3)$ b. _____
c. $(h \circ g)(2)$ c. _____
d. $(h \circ g)(1)$ d. _____
e. $(h \circ g)(-2)$ e. _____
f. Find the rule of the function $(h \circ g)(x)$. f. _____

14. Let $f(x) = x + 5$, $g(x) = 2x$, and $h(x) = x - 2$.
    a. If $k(x) = (f \circ g)(x)$, find the rule of function $k(x)$. 14.a. _____
    b. Find the rule of $(f \circ g) \circ h)(x)$, that is, the rule of $(k) \circ h)(x)$. b. _____
    c. If $r(x) = (g \circ h)(x)$, find the rule of the function $r(x)$. c. _____
    d. Find the rule of $(f \circ (g \circ h))(x)$, that is, the rule of $(f \circ (r))(x)$. d. _____
    e. Using parts b and d, does $((f \circ g) \circ h)(x) = (f \circ (g \circ h))(x)$? If yes, what group property is demonstrated? If no, explain why. e. _____

15. Let $b(x) = |x|$, $d(x) = [x]$, $f(x) = 1/x$, $g(x) = x - 3$, and $h(x) = 2x$.
    Find the rule of the given composition.
    a. $(f \circ b)(x)$ 15.a. _____
    b. $(d \circ g)(x)$ b. _____
    c. $(b \circ g)(x)$ c. _____
    d. $(g \circ d)(x)$ d. _____
    e. $(g \circ f)(x)$ e. _____
    f. $(h \circ g)(x)$ f. _____
    g. $(f \circ (g \circ h))(x)$ g. _____
    h. $(d \circ (h \circ f))(x)$ h. _____
    i. $(g \circ (h \circ b))(x)$ i. _____
    j. $(b \circ (f \circ h))(x)$ j. _____

16. To find a 6% sales tax on an item, Ms. Reres reasoned as follows: Using function s, I multiply the price of the item by .06; then, using function r, I round the amount found to the nearest whole cent.
    a. True or False: If x is the price of an item, then the 6% sales tax on the item is found by the composition $(r \circ s)(x)$. 16.a. _____
    b. Evaluate $(r \circ s)(\$1.39)$. b. _____
    c. Evaluate $(r \circ s)(\$16.79)$. c. _____
    d. True or False: If the 6% sales tax is added to the price of the item by function t, the the customer's bill is found by $(t \circ (r \circ s))(x)$. d. _____
    e. Evaluate $(t \circ (r \circ s))(\$3.15)$. e. _____
    f. Evaluate $(t \circ (r \circ s))(\$19.95)$. f. _____

Regents Review: Functions, Inverses Name: _____

1. If a function g is defined by $g(x) = 2x^2 - x$, find the value of g(-3). 1. _____

2. If the function g is defined by g(x) = 2x - 4, find the value of g(-6). 2. _____

3. Which of the following is not a function? 3. _____

(1) 3x + 2y = 7 (3) $y = 2x^2 + 3$
(2) y = 4 (4) x = 4

4. The domain for f(x) = 2x - 1 is $-3 \leq x \leq 3$. The greatest value in the range of f is: 4. _____

(1) -12 (3) 5
(2) -7 (4) 12

5. The domain of the relation $y = \frac{4}{\sqrt{x - 1}}$ is the set 5. _____

(1) $\{x \mid x > 1\}$ (3) $\{x \mid x < 1\}$
(2) $\{x \mid x > 1\}$ (4) $\{x \mid x \leq 2\}$

6. What is the domain of the function $f(x) = \sqrt{x - 1}$? 6. _____

(1) $\{x \mid x \geq 1\}$ (3) $\{x \mid x \leq 1\}$
(2) $\{x \mid x \geq 2\}$ (4) $\{x \mid x \leq -2\}$

7. What is the domain of the function $f(x) = \sqrt{x - 2}$? 7. _____

(1) $\{x \mid x \geq 1\}$ (3) $\{x \mid x \leq 2\}$
(2) $\{x \mid x \geq 2\}$ (4) $\{x \mid x \geq -2\}$

8. Which is the inverse functionof $\{(-2, 3) (7,2) (8, -11)\}$? 8. _____

(1) $\{(-1/2, 1/3) (1/7, 1/2) (1/8, -1/11)\}$
(2) $\{(2,3) (-7,2) (-8, -11)\}$
(3) $\{(3, -2) (2,7) (-11, 8)\}$
(4) $\{(-3,2) (-2, 7) (11, 8)\}$

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9. What is the inverse of the function ((-1, 3), (0,5) (3, -2))? 9. _____

(1) (1, -2), (0, -5), (-3, 2))
(2) (-1, -2), (0, -5), (3, 2))
(3) ((2, -1), (5, 0), (-2, 3))
(4) ((-2, 1), (-5, 0), (2, -3))

10. The inverse of the function $y = 2x$ is: 10. _____

(1) $x = -2y$ (3) $y = x - 2$
(2) $x = 2y$ (4) $y = -2x$

11. The inverse of the function $y = 3x + 2$ is: 11. _____

(1) $y = \frac{x - 2}{3}$ (3) $y = \frac{2x}{3}$
(2) $y = \frac{2 - x}{3}$ (4) $y = 3x - 2$

12. What is the inverse relation of the function whose equation is $y = 2x + 3$? 12. _____

(1) $y = 2x - 3$ (3) $y = 3x - 2$
(2) $\frac{x - 3}{2}$ (4) $y = x$

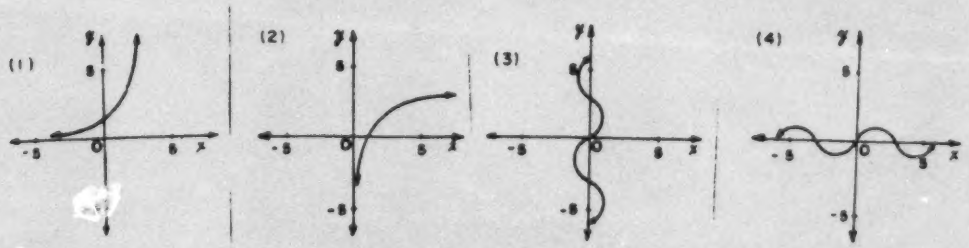
13. What is the inverse of the function $y - 2 = 3x$? 13. _____

(1) $y = \frac{2 - x}{3}$ (3) $y = 3x - 2$
(2) $y = \frac{2x}{3}$ (4) $y = \frac{x - 2}{3}$

14. The inverse of $y = 2x - 1$ is: 14. _____

(1) $y = -2x + 1$ (3) $y = \frac{1}{2}x + 1$
(2) $y = -\frac{1}{2}x - 1$ (4) $y = \frac{1}{2}x + \frac{1}{2}$

15. Which graph does not represent a function? 15. _____



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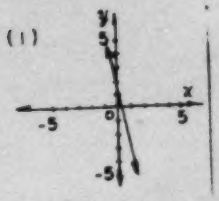
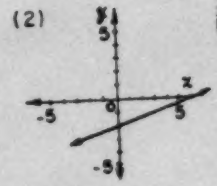
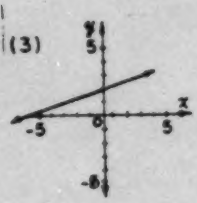
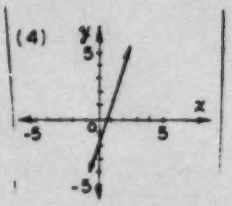
16. Which is the graph of the inverse of the function $y = 3x + 6$? 16. _____

(1) 

(2) 

(3) 

(4) 

LOGARITHMS
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Rational Exponents Name: _____

For Examples 1-6, simplify each expression.

1. a. $9^{1/3} \quad 3^{1/4} \sqrt[5]{27}$ b. $\sqrt{10} \cdot (.1)^{-1/3} \quad \sqrt[3]{100}$
2. a. $5^{\sqrt{3}} \cdot 5^{\sqrt{2}}$ b. $\left((\sqrt{2})^{\sqrt{2}}\right)^{\sqrt{2}}$
3. a. $4^{\sqrt{3}} \cdot 2^{\sqrt{3}}$ b. $3^{\sqrt{5}} \div 3^{\sqrt{2}}$
4. a. $(.01)^{\sqrt{2}} \quad 100^{\sqrt{2}}$ b. $(\sqrt{10})^{\sqrt{3}} \cdot (\sqrt{.1})^{\sqrt{3}}$
5. a. $5^{1/2} \quad \sqrt[3]{25} \quad \sqrt{.2}$ b. $(5^{\sqrt[3]{4}}) \quad \sqrt[3]{2}$
6. a. $49^{\sqrt{3}} \cdot 7^{-\sqrt{3}}$ b. $8^{\sqrt{5}} \quad (.5)^{\sqrt{5}}$

For examples 7-8, solve each equation for x.

7. a. $9^3 = 3^{x-4}$ b. $4^{3x} = 2^{\sqrt{2}}$ c. $5^{3x} = (.2)^{x-3}$
   d. $4^3 = 8^x$ e. $10^{3x} = (.01)^{x+3}$ f. $16^{2x} = 64^{x+1}$
8. a. $27^x = (1/3)^4$ b. $10^{2x} = (.10)^{x+1}$ c. $8^{\sqrt{3}} = 4^{x-1}$
   d. $125^x = 25^{x+7}$ e. $5^{4x} = (.2)^{x-2}$ f. $7^{2x} = 7^5$

Graphing Exponential Functions Name: _____

1. a. Sketch the graph of $y = 3^x$ in the interval $-3 \leq x \leq 3$.

   b. From the graph, approximate the value of $3^{2.2}$ to the nearest tenth.

2. a. Sketch the graph of $y = 4^x$ in the interval $-2 \leq x \leq 2$.

   b. From the graph, approximate the value of $4^{\sqrt{2}}$ to the nearest tenth.

3. a. Sketch the graph of $y = (\frac{1}{4})^x$ in the interval $-2 \leq x \leq 2$.

   b. For the domain of real numbers, what is the range of the funtion $y = (\frac{1}{4})^x$?

4. a. Sketch the graph of $y = (5/2)^x$ in the interval $-3 \leq x \leq 3$.

   b. Sketch the reflection in the y-axis of the graph of $y = (5/2)^x$.

   c. What is an equation of the graph drawn in answer to part b?

Multiple Choice:

5. The equation $y = a^x$ defines an exponential function when a equals:

   (1) 1 (2) 2 (3) 0 (4) -1

6. The graph $y = b^x$ where $b > 0$ and $b = 1$, lies in quadrants:

   (1) I and II (2) I and IV (3) II and III (4) II and IV

7. If $b > 0$, then the graph of $y = b^x$ must contain the point:

   (1) (-1,0) (2) (1,0) (3) (0,1) (4) (0,0)

8. The equation that defines the same function as $y = 5^x$ is:

   (1) $y = 5^{-x}$ (2) $Y = -5^{-x}$ (3) $y = (1/5)^{-x}$ (4) $y = (1/5)^x$

9. The graphs of $y = 3^x$ and $y = 3^{-x}$ intersect at:

   (1) (0,1) only (2) (1,0) only
   (3) (0,1) and (1,3) (4) no point

Logarithmic and Exponential Notations Name:_____

For examples 1 and 2, write each equation in exponential form.

1. (a) $\log_2 16 = 4$ (b) $\log_{10} 10{,}000 = 4$ (c) $.5 = \log_9 3$

   (d) $\log_x 28 = 3$ (e) $\log_6 x = -3$ (f) $x = \log_{10} 119$

2. (a) $\log_3 243 = 5$ (b) $-6 = \log_2 1/64$ (c) $2 = \log_{1/2} 1/4$

   (d) $\log_x 53 = 2$ (e) $\log_{1/4} x = -5$ (f) $x = \log_5 33$

For examples 3 and 4, write each equation in logarithmic form.

3. (a) $8^3 = 512$ (b) $3^{-4} = 1/81$ (c) $27^{1/3} = 3$

   (d) $1/128 = {}^{x-6}$ (e) $x = 7^4$ (f) $139 = 10^x$

4. (a) $6^4 = 1296$ (b) $5^{-3} = 1/125$ (c) $8^{1/3} = 2$

   (d) $1/256 = {}^{x-4}$ (e) $x = 3^8$ (f) $168 = 5^x$

For 5 - 14, find the value of x

5. $\log_3 81 = x$
6. $\log_5 125 = x$
7. $\log_3 1/27 = x$
8. $\log_4 x = 3$
9. $\log_3 x = -2$
10. $\log_{1/2} x = -3$
11. $\log_x 8 = 2$
12. $\log_x 4 = -2$
13. $\log_x 64 = 2$
14. $\log_x 2 = 1/2$

Logarithm Tables Name: _____

For examples 1 and 2, find the common logarithm of each number.

1. (a) 4,530,000 (b) .00021 (c) 35,400,000
   (d) .073 (e) .0000006 (f) 3200

2. (a) 73,600,000 (b) 202,000 (c) .0301
   (d) .000000011 (e) 27.2 (f) 956,000,000

For examples 3 and 4, find the antiogarithm of each number.

3. (a) 5.7679-10 (b) 6.2900-10 (c) 6.6646
   (d) 1,3856 (e) 7.9754-10 (f) 2.0000

4. (a) 4.6493-10 (b) 2.8096-10 (c) 2.1173
   (d) 5,7267 (e) 1.0000 (f) 6.8089-10

For examples 5 and 6, use interpolation to find the logarithm of each number.

5. (a) 78,340 (b) 138.6 (c) .8183
   (d) 14.14 (e) .006158 (f) 2836

6. (a) 5.113 (b) 469.7 (c) .08532
   (d) 6354 (e) 31,480 (f) 68.57

For examples 7 and 8, use interpolation to find the antilogarithm of each number.

7. (a) 5,3169 (b) 7,1128 - 10 (c) 8.4739 - 10
   (d) 2.2212 (e) 6.4875 - 10 (f) 1.9762

8. (a) 4.6164 (b) 8.2023 - 10 (c) 7.7369 - 10
   (d) 2.7742 (e) 5.8126 - 10 (f) 1.2991

Calculations Using Logs Name: _____

For examples 1-8, find x.

1. $\log 3 + \log 8 = \log x$
2. $\log 5 + \log x = \log 30$
3. $\log 18 - \log 3 = \log x$
4. $\log 36 = x \log 6$
5. $(\log 2 + \log 12) - \log 6 = \log x$
6. $(\log 10 - \log 2) + \log x = \log 15$
7. $\log 27 = x \log 3$
8. $1/2 \log x = \log 7$

For examples 9-28, use the logarithm table and compute each product or quotient to four significant digits.

9. $(632)(787)$
10. $(963)(876)$
11. $(7370)(684)$
12. $(457)(9860)$
13. $(43.8)(72.9)$
14. $(50.9)(-79.6)$
15. $(-.815)(.0632)$
16. $(-.0743)(.762$
17. $(.00489)(-.0683)$
18. $(4.56)(.00987)$
19. $(603{,}000)(429.000)$
20. $(73{,}500)(529{,}000)$
21. $953 \div 46.1$
22. $7340 \div 962$
23. $-3.41 \div .0620$
24. $75.8 \div 6.92$
25. $65.4 \div 3.85$
26. $235{,}000 \div -4620$
27. $.0352 \div .00714$
28. $.0105 \div .732$

For examples 29-42, compute each power or root to four significant digits.

29. $(92.5)^4$
30. $(6.58)^3$
31. $(984)^5$
32. $(646)^4$
33. $(4.06)^3$
34. $(72.8)^4$
35. $\sqrt[3]{4.15}$
36. $\sqrt[5]{3.98}$
37. $\sqrt{7340}$
38. $\sqrt{.00529}$
39. $\sqrt[3]{5.29^2}$
40. $\sqrt[3]{36.8^2}$
41. $(3.02)^2$
42. $(7.98)^{1/3}$

For examples 43-54, compute each answer to four significant digits.

43. $\dfrac{(38)^2(56)}{189}$
44. $\dfrac{(65)(231)^2}{496}$
45. $\dfrac{(318)(765)}{(87)^2}$
46. $\dfrac{7.14\sqrt{3.51}}{2.06}$
47. $\dfrac{6.14\sqrt{11.8}}{12.9}$
48. $\dfrac{(76)^3}{(19)^4}$
49. $\dfrac{-19.6\sqrt[3]{24.6}}{(2.03)^3}$
50. $(6.23)(1.08)^7$
51. $\dfrac{596}{16.7\sqrt{23.2}}$
52. $\dfrac{8.15\sqrt[4]{13.9}}{.67}$
53. $\sqrt[5]{\dfrac{(52.8)^2}{(7.9)^2(3.2)^3}}$
54. $\sqrt[5]{\dfrac{(28.1)(13.4)^2}{(7.28)^3}}$

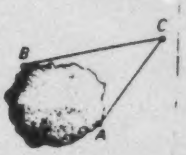
Solving Triangles With Logs Name: _____

For examples 1-8, use logarithms to find the unknown measures of all triangles satisfying the given conditions or state tha no triange is possible.

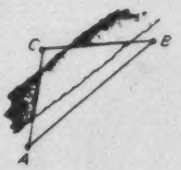
1. a = 5390, = 37^{o} 40', B = 82^{0} 50'
2. c = 974, B = 43^{0} 10' , γ = 65^{0} 20'
3. a = 95.45, b = 67.15, γ = 35^{0} 10'
4. a = 580, b = 8541, c = 8243
5. a = 533, b = 825, c = 35^{0} 22'
6. a = 2249, b = 5348, B = 45^{0} 21'
7. c = 4[illegible]0, β = 38^{0} 15', γ = 25^{0} 29'
8. b = 8965, B = 75^{0} 21' , a = 33^{0} 17'

For examples 9-12, use logarithms wherever it is advantageous.

9. To find the distance from A to B across a pond, a man selected a point C such that $\overline{AC}$, $\overline{BC}$, and $\angle C$ could be measured. He found that AC = 749 feet, BC = 853 feet, and m $\angle C$ = 44^{0} 25'. What is the distance from A to B?

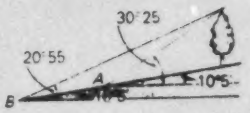


10. Points A and B on one side of a river are 467 feet apart. Point C on the other side of the river is such that m $\angle$ CAB = 64^{0} 13' and m $\angle$ CBA = 35^{0} 15'. What is the distance
a. from C to A? b. from C to B?



11. A tree grows vertically on a slope inclined an an angle of 9^{0} to the horizontal. The tree casts a shadow 249 feet directly down the hill when the angle of elevation of the sum is 28^{0}. How tall is the tree?

12. A tree grows vertically on a slope included at an angle of 10^{0}5' t o the horizontal. From Point A the angle of elevation of the top of the tree is 30^{0}25'. From point B, 100 feet farther down the slope, the angle of elevation of the top of the tree is 20^{0}55'. If A,B, and the base of the tree lie on a line, how tall is the tree?



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Regent's - Logs and Exponents Name: _____

1. Find the value of log 37.62 to four decimal places. 1. _____

2. If $f(x) = x^{-\frac{1}{2}}$, find f(4). 2. _____

3. Find the numerical value of $8^{2/3} + 4^0$. 3. _____

4. If $f(x) = x^{2/3}$, find f(-27). 4. _____

5. Solve for x: $9^{2x} = 3^{3x+1}$. 5. _____

6. Solve for x: $2^{3x+2} = 4^{2x}$. 6. _____

7. Evaluate $(a^{-1})^{2/3}$ where a = 8.

8. If log N = log A + 3 log B, solve for N in terms of A and B. 8. _____

9. If $\log_x 3 = 1/3$, find x. 9. _____

10. If $\log_9 x = 1/2$, find x. 10. _____

11. If log a = 0.4262, find $\log \sqrt{a}$. 11. _____

12. If $\log_9 81 = x$, find x. 12. _____

Multiple Choice:

13. The solution set of $3^{2x-1} = 3^{x2}$ is 13. _____
(1) {1} (3) {1 - 1}
(2) {-1} (4) { }

14. The graph of $y = (\frac{1}{2})^x$ lies entirely in quadrants 14. _____
(1) I and II (3) II and III
(2) I and III (4) I and IV

15. If log 4.72 = m, then log 472 equals 15. _____
(1) m + 2 (3) 100m
(2) 2m (4) 100 + m

16. If log m = 1.2970, then log 100m equals 16._____
(1) 129.7 (3) 3.2970
(2) 12.97 (4) 8.2970 - 10

17. Which is equivalent to $\log\sqrt{a/b}$? 17._____
(1) (log a - log b) (3) $\frac{\log a}{\log b}$
(2) 1 (log a - log b) (4) $\sqrt{\frac{\log a}{\log b}}$

18. Which statement is true for the graph of $y = \log_{10} x$? 18. _____
(1) It is an increasing function and lies entirely in Quadrants I and II.
(2) It is an increasing function and lies entirely in Quadrants I and IV.
(3) It is a descreasing function and lies entirely in Quadrants I and II.
(40 It is a decreasing function and lies entirely in Quadrants I and IV.

SHOW ALL WORK:

19. Using logarithms, solve for x to the nearest tenth. 19. _____

$3^{2x} = 5$

20. Using logarithms, find $\sqrt[3]{0.692}$ to the nearest hundredth. 20. _____

21. a. Sketch the graph of $F = \{(x,y) \mid y = 2^x\}$.
b. On the same set of axes, sketch the graph of F^{-1} by reflecting the graph drawn in part a over the line $y = x$.
c. Write an equation for F^{-1}.

Regent's - Logs and Exponents Name: _____

1. If $f(x) = \log x$, find $f(8)$. 1. _____

2. Find the value of log 429.7 coreect to <u>four decimal places</u>. 2. _____

3. Evaluate: $a^0 \div a^{-\frac{1}{2}}$, when $a = 9$. 3. _____

4. If $10^{0.4771} = 3$, then what is th value of $10^{2.4771}$? 4. _____

5. If $a = 4$, find the value of $(4a^0)^{\frac{3}{2}}$. 5. _____

6. If $f(x) = (3x)^{-2}$, find $f(2)$. 6. _____

7. If $\log_4 x = 3$, find x. 7. _____

8. Solve for x: $27^x = 3^{2x-1}$. 8. _____

9. If $\log_x 125 = 3$, find x. 9. _____

10. If a function is define by $f(x) = \log_{10} x$, find the value of $f(10)$. 10. _____

11. Solve for x: $81^x = 9^{x+1}$. 11. _____

12. The expression $\log\sqrt{xy}$ is equivalent to 12. _____

(1) 2 log x log y (3) ½log x log y
(2) 2 (log x + log y) (4) ½(log x and + log y)

13. The graph of $y = \log_{10} x$ lies entirely in which quandrants? 13. _____

(1) I and II (3) I and III
(2) I and IV (4) III and IV

14. The solution set of $2^{x2+2x} = 2^{-1}$ is 14. _____

(1) $\{1\frac{1}{2}\}$ (3) $\{1, -1\frac{1}{2}\}$
(2) $\{-1\frac{1}{2}\}$ (4) $\{\frac{1}{2}, \frac{1}{2}\}$

15. If $x = \frac{a\sqrt{b}}{c}$, the log x is equal to 15. _____

(1) log a + ½ log b - log c
(2) log a + 2 log b - log c
(3) log a = ½ log b + log c
(4) log a - 2 log b - log c

16. The expression log a - ½ log b is equivalent to 16. _____

(1) $\frac{\log a}{\frac{1}{2}\log b}$ (3) $\log \frac{a}{\sqrt{b}}$

(2) $\frac{\log a}{\log b}$ (4) $\log \frac{a}{b^2}$

17. If log 2 = A and log 3 = B, then log 6 is to 17. _____

(1) A + B (3) AB
(2) A - B (4) $\frac{A}{B}$

18. The graph of $y = a^2$, where a O, contains the point 18. _____

(1) (1,0) (3) (0,0)
(2) (-1,0) (4) (0,1)

19. Using logarithms, find N to the nearest hundredth: $N = 3.14\sqrt{.432}$ 19. _____

20. Using logarithms, find N to the nearest hundredth: $N = \sqrt[3]{.875}$ 20. _____

21. Using logarithms, solve the equation $3^x = 21$ for x to the nearest tenth. 21. _____

Regent's Logs and Exponents

Name: _____

1. If $f(x) = 4x^{\frac{1}{2}}$, find $f(4)$. 1. _____

2. If $f(x) = \log (x^2)$, find $f(10)$. 2. _____

3. Find the value of $16^{3/4}$. 3. _____

4. If $m = 8$, find the value of $m^{2/3}$. 4. _____

5. Solve for x in tems of a and b: 5. _____
   $\log_b x = a$

6. Evaluate $m^{\frac{3}{2}}$ when $m = 4$. 6. _____

7. Solve for x: $3^{2x+1} = 9$. 7. _____

8. Solve for x: $3^{3x-1} = 3^{2x+2}$. 8. _____

9. Find the value of $4a^{-\frac{1}{2}}$ if $a = 9$. 9. _____

10. Evaluate $x^{2/3} + x^{-1}$ when $x = 8$. 10. _____

11. Find $\log \sqrt[3]{.691}$ to <u>four decimal places</u>. 11. _____

12. If $\log_3 n = 4$, what is the value of n? 12. _____

13. The expression $\log \sqrt[3]{\frac{a}{b}}$ is equivalent to 13. _____

    (1) 1/3 (log a - log b)
    (2) 1/3 log a + log b
    (3) 1/3 $\left(\frac{\log a}{\log b}\right)$
    (4) 3(log a - lob b)

14. If $10^{0.9370} = 8.65$, then $10^{2.9370}$ is equal to 14. _____

    (1) 865 (3) 17.3
    (2) 86.5 (4) 0.0865

Regents - Logs and Exponents - Multiple Choice

15. If $10^x = m$, then 10^{3x} equals 15. _____

(1) 3m (3) 3^m
(2) 3 + m (4) m^3

16. The expression $\log\sqrt{xy}$ is equivalent to 16. _____

(1) 2 log x log y (3) ½ log x log y
(2) 2 (log x + log y) (4) ½(log x + log y)

17. If the graphs of y = 1 and y = log x are sketched on the same axes, then the number of points in their intersection is 17. _____

(1) 1 (3) 3
(2) 2 (4) 0

18. Which value satisfies the equation $9^x = 27$? 18. _____

(1) -2 (3) -3/2
(2) 2 (4) 3/2

19. Using logarithms, solve for x to the nearest tenth: 19. _____

$2^x = 5$

20. Using logarithms, find x to the nearest tenth: 20. _____

$4^x = 12$.

21.a. Using logarithms, find the value of $\log_3 4$ to the nearest tenth. 21.a. _____

21.b. If $N = \sqrt[4]{\frac{7^3}{6}}$, which is equivalent to log N? 21.b. _____

(1) 3 (log 7 - ¼ log 6)
(2) 1/3 log 7 - 4 log 6
(3) $\frac{3 \log 7}{\frac{1}{4} \log 6}$
(40 3 log 7 - ¼ log 6

c. The expression $\log b^{m+1}$ is equivalent to 21.c. _____

(1) 10 log b^m
(2) m log b + log b
(3) m log b + b
(4) m log b

TRIGONOMETRY
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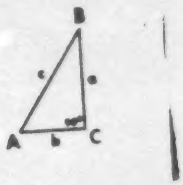
Right Triange Trig - Use of Definitions Name: _____

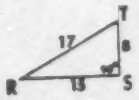
Exercises 1-7 refer to right △ ABC. Name the ratio that can be used to find:

1. side a when angle A and side c are given
2. side b when angle A and side c are given
3. side c when side a and angle A are given
4. side b when side a and angle B are given
5. angle A when side a and side b are given
6. angle B when side a and side c are given
7. side a when side b and angle A are given



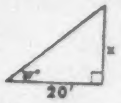
Exercises 8-13 refer to △ RST. In each exercise, give the value of the ratio as a fraction.

8. sin R
9. tan T
10. sin T
11. cos R
12. cos T
13. tan R



In 14-20 find the line marked x to the nearest foot or the angle marked x to the nearest degree.

14.



15.

16.

Right Triangle Trig - Use of Definitions

17.

18.

19.

20.

Right △ Trig - Word Problems Name: _____

1. A monument stands on level ground. The angle of elevation of the top of the monument, taken at a point 425 feet from the foot of the monument, is 32^0. Find the height of the monument to the nearest foot.

2. A boy flying a kit lets out 140 feet of string that makes an angle of 64^0 with the ground. Assuming that the string is straight, find to the nearest foot how high the kite is above the ground.

3. A 25-foot guy wire attached to the top of a pole makes an angle of 62^0 with the ground. Find to the nearest foot the distance between the point where the guy wire meets the ground and the foot of the pole.

4. A boy walked 400 feet into a tunnel which slopes downward at an angle of 7^0 with the horizontal ground. Find to the nearest ten feet how far he was beneath the surface.

5. An airplane A is 1000 feet above the ground and directly over a church C. The angle of elevation of the plane as seen by a boy at a point B on the ground some distance from the church is 22^0. Fine to the nearest foot:
   a. how far the boy is from the church
   b. how far the boy is from the plane

6. An observer in a balloon which is 2000 feet above an airport finds that the angle of depression of a steamer out at sea is 21^0. Find to the nearest hundred feet the distance between the balloon and the streamer.

7. A plane takes off from a field and climbs at an angle of 12^0. Find to the nearest 100 feet how far the plane must fly to be at an altitude of 1200 feet.

8. Find to the nearest degree the angle of elevation of the sun when a tree 24 feet high casts a shadow of 36 feet.

9. A 40-foot ladder is leaning against a building. The foot of the ladder is 32 feet from the building. Find to the nearest degree the angle which the ladder makes with the ground.

10. If a tree 35 feet tall casts a shadow 45 feet long, what is the angle of elevation of the sun to the nearest degree?

Angle Reduction and Quadrants Name: _____

For examples 1 and 2, give the value of each expression in temrs of a number between 0 and 45.

1. a. $\sin 67^0$ b. $\cos 52^0$ c. $\cot 58^0$ d. $\sec 84^0$
   e. $\csc 77^0$ f. $\tan 76^0$ g. $\cos 56^0$ h. $\sin 87^0$

2. a. $\sin 78^0$ b. $\cos 81^0$ c. $\cot 63^0$ d. $\sec 49^0$
   e. $\csc 54^0$ f. $\tan 72^0$ g. $\cos 74^0$ h. $\sin 67^0$

For examples 3 and 4, give the value of each expression in terms of a number between 0 and 90.

3. a. $\cos 113^0$ b. $\cot 195^0$ c. $\sec 145^0$ d. $\tan 286^0$
   e. $\sin 317^0$ f. $\csc 208^0$ g. $\cos 103^0$ h. $\cot 352^0$

4. a. $\sec 149^0$ b. $\tan 194^0$ c. $\sin 316^0$ d. $\cos 273^0$
   e. $\cot 212^0$ f. $\csc 328^0$ g. $\cos 121^0$ h. $\tan 237^0$

For each value of a, classify it as a first, second, third, or fourth quadrant number.

a. $a = 145$ b. $a = 220$
c. $a = 340$ d. $a = 820$
e. $a = -145$ f. $a = -220$
g. $a = -340$ h. $a = -820$

6. For each value of a in Ex. 5, tell whether the following are positive or negative.

   a. $\sin a^0$ b. $\cos a^0$ c. $\tan a^0$

   d. $\csc a^0$ e. $\sec a^0$ f. $\cot a^0$

Interpolation Name: _____

For examples 1-4, use the table to find the value of each expression.

1. a. $\cos 35^0\ 48'$ b. $\sin 78^0\ 59'$ c. $\sec 10^0\ 11'$
   d. $\tan 55^0\ 37'$ e. $\cot 10^0\ 26'$ f. $\csc 45^0\ 3'$

2. a. $\sin 24^0\ 12'$ b. $\tan 8^0\ 44'$ c. $\csc 287^0\ 13'$
   d. $\cos 33^0\ 18'$ e. $\sec 57^0\ 17'$ f. $\cot 46^0\ 13'$

3. a. $\sin -(37^0\ 15')$ b. $\tan 185^0\ 12'$ c. $\csc 287^0\ 13'$
   d. $\cot 188^0\ 17'$ e. $\sec 254^0\ 18'$ f. $\cos -(368^0 19')$

4. a. $\sin -(145^0\ 15')$ b. $\tan -(33^0\ 11')$ c. $\csc 271^0\ 18'$
   d. $\cot -(145^0\ 17')$ e. $\sec 123^0\ 13'$ f. $\cos 756^0\ 17'$

For examples 5-8, use the table to find 0 or t.

5. a. $\sin \theta^0 = .7815$ b. $\tan \theta^0 = 1,3467$ c. $\sec \theta^0 = 1.176$
   c. $\csc \theta^0 = 2.478$ e. $\cos \theta^0 = .9010$ f. $\cot \theta^0 - .8850$

6. a. $\tan \theta^0 - .1532$ b. $\cot \theta^0 = 22.20$ c. $\sin \theta^0 = .9624$
   d. $\cos \theta^0 = .8714$ e. $\sec \theta^0 = 1.220$ f. $\csc \theta^0 = 2.430$

7. a. $\cot t = -.16050$ b. $\cos t = .71701$ c. $\sin t = .60511$
   d. $\sin t = .96725$ e. $\tan t = .23425$ f. $\cos t = .07075$

8. a. $\sin t = .41108$ b. $\text{cost } t = -.26596$ c. $\tan t = .54730$
   d. $\cot t = .95100$ e. $\sin t = .93810$ f. $\cos t = .78390$

Degree/Radian Measures Name: _____

Convert each degree measure in Example 1 and 2 to radian measure (using π).

1. a. 30 b. 15 c. 130 d. -60
   e. 210 f. 400 g. 450 h. 315

a. _____
b. _____
c. _____
d. _____
e. _____
f. _____
g. _____
h. _____

2. a. 45 b. 75 c. 140 d. -15
   e. 240 f. 390 g. 540 h. 330

a. _____
b. _____
c. _____
d. _____
e. _____
f. _____
g. _____
h. _____

Convert each radian measure in Example 3 and 4 to a degree measure.

3. a. $\pi/4$ b. $2\pi/3$ c. $5\pi/12$ d. $7\pi/6$
   e. 2 f. -1.5 g. 3π h. $5\pi/2$

a. _____
b. _____
c. _____
d. _____
e. _____
f. _____
g. _____
h. _____

Degree/Radian Measures

4. a. $\pi/3$ b. $3\pi/4$ c. $13\pi/12$ d. $5\pi/6$

e. -1 f. $.5$ g. 4π h. $7\pi/2$

a. _____

b. _____

c. _____

d. _____

e. _____

f. _____

g. _____

h. _____

Inverse Functions Name: _____

In 1-6, rewrite the statement as an equivalent statement, using arc sin, arc cos, or arc tan.

1. $\sin O = \frac{1}{2}$
2. $\cos O = \frac{\sqrt{3}}{2}$
3. $\tan \theta = 2$
4. $\sin O = .1$
5. $\tan O = -4/5$
5. $\cos O = 1$

In 7-12, rewrite the statement as an equivalent statement, using sin O, cos O, or tan O.

7. $O = \text{arc cos } \frac{1}{2}$
8. $O = \text{arc tan } (-\sqrt{3})$
9. $O = \text{arc sin } \frac{\sqrt{2}}{2}$
10. $O = \text{arc tan } 1$
11. $O = \text{arc sin } (-1)$
12. $\theta = \text{arc cos } O$

In 13-29, find all values of O in radians for $O \leq 0 \leq 2$.

13. $O = \text{arc sin } (-\frac{1}{2})$
14. $O = \text{arc cos } \frac{\sqrt{3}}{2}$
15. $O = \text{arc tan } \sqrt{3}$
16. Find sin (arc cos $\frac{1}{2}$)
17. Find cos (arc sin (-1))
18. Find sin (arc csc 4)
19. Find th positive value of cos sin 4/5)
20. Find the positive value of sin (cos (-5/13))
21. Find the positive value of cos (arc sin .28)
22. Express cos (arc sin a) in terms of a.
23. Express sin (arc sin b) in terms of b.
24. $O = \text{arc tan } (-1)$
25. $O = \text{arc sin } \frac{\sqrt{3}}{2}$
26. $O = \text{arc cos } (-\frac{\sqrt{2}}{2})$
27. Find tan (arc sin O)
28. Find sec (arc cos .3)
29. Fin tan (arc cot 3/2)

In 30-33, select the numberal preceding the expression that best completes the sentence or answers the question.

30. If $x = \sin \text{arc } (-\frac{1}{2})$, then x can be equal to:

(1) 30^0 (2) 60^0 (3) $300^)$ (4) $330^)$

31. If $y = \text{arc cos } (-\frac{1}{2})$, then y can be equal to:

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

32. If $O = \text{arc sin } \sqrt{2}$, then θ can <u>not</u> equal:

(1) 45^0 (2) 135^0 (3) 405^0 (4) 585^0

33. If $\theta = \text{arc cos } 1$, then θ can <u>not</u> equal which radian measure?

(1) 0 (2) π (3) 2π (4) -4π

Inverse functions Name: _____

In 1-12, find the value of θ in degrees.

1. θ = Arc cos $\frac{1}{2}$
2. θ = Arc sin 0
3. θ = Arc tan 1
4. θ = Arc sin $\frac{1}{2}$
5. θ = Arc cos (-1)
6. θ = Arc cos $\left(-\frac{\sqrt{3}}{2}\right)$
7. θ = Arc tan $(-\sqrt{3})$
8. θ = Arc sin (-1)
9. θ = Arc tan 0.
10. θ = Arc sin $\left(-\frac{\sqrt{2}}{2}\right)$
11. θ = Arc cos 0
12. θ = Arc cos 1

In 13--24, find the value of 0 in radians.

13. θ = Arc sin $\frac{\sqrt{3}}{2}$
14. θ = Arc tan $\sqrt{3}$
15. θ = Arc cos $\left(-\frac{\sqrt{2}}{2}\right)$
16. θ = Arc sin 1
17. θ = Arc sin $(-\frac{1}{2})$
18. θ = Arc tan (-1)
19. θ = Arc cos $\frac{2}{2}$
20. θ = Arc tan $\left(-\frac{\sqrt{3}}{3}\right)$
21. θ = Arc cos $(-\frac{1}{2})$
22. θ = Arc sec 2
23. θ = Arc cot $(-\sqrt{3}$
24. θ = Arc csc $\sqrt{2}$

In 25-33, find the value of the given expression.

25. sin (arc cos (-1))
26. tan (Arc sin 0)
27. tan (Arc cos 4/5)
28. cos (Arc tan (-5/12))
29. sec (Arc cos 1/6)
30. sin (Arc csc $(-\sqrt{2})$)
31. cot (Arc tan 1)
32. cos (Arc sin 1/3)
33. sec (Arc tan $\sqrt{3}$)

In 34-41, select the numeral preceding the expression that best completes the sentence.

34. If θ = Arc sec $\sqrt{2}$, then θ equals:

    (1) 30^0 (2) 45^0 (3) 60^0 (4) 90^0

35. The value of $\frac{1}{2}$ cot (Arc tan 8/5) is:

    (1) 5/4 (2) 5/16 (3) 4/5 (4) 8/5

Inverse Functions

36. The value of Arc sin $(-\frac{1}{2})$ is:

(1) 30^0 (2) -60^0 (3) -30^0 (4) 210^0

37. If θ = Arc cos $\left(-\frac{\sqrt{3}}{2}\right)$, then θ equals:

(1) $-\frac{\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $-\frac{5\pi}{6}$ (4) $\frac{5\pi}{6}$

38. A value of y that is <u>not</u> in the range of the function y = Arc sin x is:

(1) 0 (2) $\frac{\pi}{2}$ (3) π (4) $-\frac{\pi}{2}$

39. The value of 2 cos (Arc sin (-.6) is:

(1) 1.6 (2) -1.6 (3) .8 (4) -.8

40. If f(x) = Arc sin x, the $f(-\frac{1}{2})$ is:

(1) $\frac{\pi}{3}$ (2) $-\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $-\frac{\pi}{6}$

41. A number that is <u>not</u> an element of te range of y = Arc tan x is:

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) 0 (4) $-\frac{\pi}{6}$

Sine Graphs Name: _____

For each function, (a) find its amplitude and period (b) find a location of its maximum valve and a location of its minimum valve (c) sketch the graph for each function in the interval -2π to 2π.

1. $y = \sin x$
2. $y = \sin x$
3. $y = \sin 2x$
4. $y = \sin(-x)$
5. $y = -2\sin x$
6. $y = \sin \frac{1}{2}x$
7. $y = 2/3 \sin x$
8. $y = 3 \sin 2x$
9. $y = 2 \sin \frac{1}{2}x$
10. $y = -2 \sin \frac{1}{2}x$
11. $y = 2 \sin 1/3x$
12. $y = -\sin(-2x)$
13. $y = 3 \sin \frac{1}{2}x$
14. $y = \frac{1}{4}\sin 2x$
15. $y = -2 \sin 1/3\ x$

Cosine Graphs Name: _____

For each function (a) find its amplitude and period, (b) find a location of its maximum valve and a location of its minimum valve, (c) sketch the graph for each function in the interval -2π to 2π .

1. $y = \cos x$
2. $y = -\cos x$
3. $y = \cos 2x$
4. $y = \cos x\ (-x)$
5. $y = -2 \cos x$
6. $y = \cos \frac{1}{2}x$
7. $y = 2/3 \cos x$
8. $y = 3 \cos 2x$
9. $y = -2 \cos \frac{1}{2}x$
10. $y = 2 \cos \frac{1}{2} x$
11. $y = 2 \cos 1/3\ x$
12. $y = -\cos (-2x)$
13. $y = 3 \cos \frac{1}{2}x$
14. $y = \frac{1}{3} \cos 2x$
15. $y = -2 \cos 1/3\ x$

Tangent Graphs Name: _____

For each function (a) find its period and (b) sketch the graph in the inverval -2π to 2π.

1. $y = \tan x$
2. $y = \tan 2x$
3. $y = -\tan x$
4. $y = 2 \tan x$
5. $y = \tan (-x)$
6. $y = -\tan 3x$
7. $y = 1/3 \tan x$
8. $u = 2 \tan \frac{1}{2}x$
9. $y = \tan \frac{1}{2} x$
10. $y = \frac{1}{2} \tan 2x$

Sum and Difference Formulas - Sine Name: _____

In 1-6, express the given sine value in terms of sin θ, or cos θ, or both.

1. $\sin\left(\frac{\pi}{2} + \theta\right)$ 2. $\sin(0 - \pi)$ 3. $\sin\left(\frac{\pi}{4} + \theta\right)$

4. $\sin\left(\theta - \frac{\pi}{2}\right)$ 5. $\sin\left(\frac{\pi}{4} - \theta\right)$ 6. $\sin(\pi + \theta)$

7. If $\sin x = 4/5$, $\cos y = 4/5$, and s and y are measures of angles in the first quadrant, find the value of $\sin(x + y)$.

8. If $\angle B$ is acute and $\sin B = 12/13$, find the value of $\sin(90^0 - B)$.

In 9-14, select the numeral preceding the expression that best completes the sentence.

9. The expression $\sin 40^0 \cos 15^0 + \cos 40^0 \sin 15^0$ is equivalent to:

(1) $\sin 55^0$ (2) $\sin 25^0$ (3) $\cos 55^0$ (4) $\cos 25^0$

10. The expression $\left(\frac{\pi}{6} - x\right)$ is equivalent to:

(1) $\frac{1}{2} - \sin x$ (2) $\frac{\sqrt{3}}{2} - \sin x$

(3) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$ (4) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

11. If $\sin(A - 30^0) = \cos 60^0$, the number of degrees in the measure of angle A is:

(1) 30 (2) 60 (3) 90 (4) 120

12. If x and y are the measures of positive acute angles, $\sin x = \frac{1}{2}$, and $\sin y = 4/5$, then $\sin(x + y)$ equals:

(1) $\frac{3 + 4\sqrt{3}}{10}$ (2) $\frac{3 - 4\sqrt{3}}{10}$ (3) $\frac{\sqrt{3}}{4} + \frac{12}{25}$ (4) $\frac{\sqrt{3}}{4} - \frac{12}{25}$

13. If $\sin x = \frac{-1}{3}$, $\sin y = -\frac{\sqrt{5}}{3}$, and x and y are the measures of angles in the third quadrant, find: a. $\sin(x + y)$ b. $\sin(x - y)$ c. $\sin(y - x)$

14. If $\sin A = 3/5$, $\angle A$ is in quadrant I, $\cos B = 5/13$, and $\angle B$ is in quadrant II, find: a. $\sin(A + B)$ b. $\sin(A - B)$ c. $\sin(B - A)$ d. $\cos(A + B)$ e. $\cos(A - B)$ f. $\cos(B - A)$

Sum and Difference Formulas - Sine

In 15-20, use the identity for the sine of the sum or difference of two angle measures to find the exact value of the given function.

15. $\sin 150^0 = \sin (60^0 + 90^0)$

16. $\sin 90^0 = \sin (135^0 - 45^0)$

17. $\sin 75^0 = \sin (45^0 + 30^0)$

18. $\sin 75^0 = \sin (12-^0 - 45^0)$

19. $\sin 105^0 = \sin (135^0 - 30^0)$

20. $\sin 105^0 = \sin (60^0 + 45^0)$

Sum and Difference Formulas - Cosine Name:_____

1. Find $\cos 105^0$ by using $\cos (45^0 + 60^0)$.
2. Find $\cos 255^0$ by using $\cos (210^0 + 45^0)$.
3. Find $\cos 135^0$ by suing $\cos (135^0 + 60^0)$.
4. Find $\cos 105^0$ by suing $\cos (135^0 - 30^0)$.
5. Find $\cos 75^0$ by using $\cos (120^0 - 45^0)$.
6. Find $\cos 165^0$ by using $\cos (225^0 - 60^0)$.
7. Find $\cos (-15^0)$ by using $\cos (30^0 - 45^0)$.
8. If $\sin A = 4/5$, $\sin B = 4/5$, $\angle A$ is in quadrant II, and $\angle B$ is in quadrant I, find the value of $\cos (A - B)$.
9. If $\sin A = 12/13$, $\angle A$ is in quadrant III, $\sin B = 4/5$, and $\angle B$ is in quadrant II, find: a. $\cos (A - B)$ b. $\cos (B + A)$
10. If $A = -\frac{1}{2}$, $\cos B = -\frac{1}{2}$, and both $\angle A$ and $\angle B$ are in quadrant III, find: a. $\cos (A - B)$,B. $\cos (B + A)$
11. If $\sin x = 3/5$ and x is the measure of a positive acute angle, find the value of $\cos (X + 180^0)$.
12. If $\sin A = 5/13$, $\sin B = 5/13$, $\angle A$ is in quadrant II, and $\angle B$ is in quadrant I, find the value of $\cos (A - B)$.

In 13-20, select the numberal preceding the expression that best completes the sentence.

13. The expression $\cos 30^0 \cos 12^0 + \sin 30^0 \sin 12^0$ is equivalent to:

    (1) $\cos 42^0$ (2) $\cos 18^0$

    (3) $\cos 42^0 + \sin 42^0$ (4) $\cos^2 42^0 + \sin^2 42^0$

14. The expression $\cos (\pi - x)$ is equivalent to:

    (1) $\sin X$ (2) $-\sin x$ (3) $\cos x$ (4) $-\cos x$

15. If $\cos (A - 30^0) = \frac{1}{2}$, then measure of angle A may be:

    (1) 30^0 (2) 60^0 (3) 90^0 (4) 120^0

16. The value of $\cos 45^0 \cos 15^0 + \sin 45^0 \sin 15^0$ is:

    (1) 1 (2) $\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 0

Sum and Difference Formulas - Cosine

17. If $\sin(90^0 - x) = \frac{1}{2}$, then $\cos x$ is equal to:

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) 0

18. If $\sin(-A) = \frac{3}{5}$, then $\sin A$ is equal to:

(1) $\frac{4}{5}$ (2) $-\frac{4}{5}$ (3) $\frac{3}{5}$ (4) $-\frac{3}{5}$

19. The expression $\cos(90^0 \pm \theta)$ is equivalent to:

(1) $\sin\theta$ (2) $-\sin\theta$ (3) $\cos\theta$ (4) $-\cos 0$

20. The value of $(\cos 67^0 30')(\cos 22^0 30') - (\sin 67^0 30')(\sin 20^0 30')$ is:

(1) 1 (2) $\frac{\sqrt{2}}{2}$ (3) $-\frac{\sqrt{2}}{2}$ (4) 0

Sum and Difference Formulas - Tangent Name: _____

1. If tan A = 4 and tab B = 3, find: a. tan (A + B) b. tan (A - B)

2. If tan θ = -6 and tan ϕ = ½, find: a. tan θ + ϕ) b. tan θ - ϕ)

3. If tan x = -2/3, and tan y = -9/4, find a. tan (x = y) b. tan (x - y)

4. Let tan A = 2 and tan B = 3. a. Find the value of tan (A + B).
   b. One possible measure of the angle (A = B) is:

   (1) 0^0 (2) 45^0 (3) 90^0 (4) 135^0

In 5-8, use the identity for tan (A + B) or for tan (A - B) to find the exact value of the given function. Rationalize the denominator of the answer.

5. $\tan 75^0 = \tan (30^0 + 45^0)$

6. $\tan 15^0 = \tan (45^0 - 30^0)$

7. $\tan 105^0 = \tan (60^0 + 45^0)$

8. $\tan 195^0 = \tan (135^0 + 60^0)$

In 9-12, select the numberal preceding the expression that best completes the sentence.

9. The expression $\frac{\tan 40^0 + \tan 30^0}{1 - \tan 40^0 \tan 30^0}$ is equivalent to:

   (1) $\tan 70^0$ (2) $\tan 10^0$ (3) $\frac{\tan 40^0}{1 - \tan 40^0}$ (4) $\frac{\tan 70^0}{1 - \tan 70^0}$

10. The expression tan (x + y) is undefined when:

    (1) tan x tan y = o (2) tan x tan y = 1
    (3) tan x tan y = -1 (4) tan x and tan y = 0

11. The expression tan (A - B) is undefined when tan a = ½ and tan B equals:

    (1) 1 (2) 2 (3) -2 (4) 0

12. Since tan 165^0 = tan (135^0 + 30^0), the exact value o tan 165^0 can be found to be:

    (1) $-2 + \sqrt{3}$ (2) $-2 - \sqrt{3}$ (3) $2 + \sqrt{3}$ (4) $2 - \sqrt{3}$

Tangent Graphs

Name: _____

For each function (a) find its period and (b) sketch the graph in the inverval -2π to 2π.

1. $y = \tan x$
2. $y = \tan 2x$
3. $y = -\tan x$
4. $y = 2 \tan x$
5. $y = \tan \frac{1}{4}(-x)$
6. $y = -\tan 3x$
7. $y = 1/3 \tan x$
8. $u = 2 \tan \frac{1}{2}x$
9. $y = \tan \frac{1}{2} x$
10. $y = \frac{1}{2} \tan 2x$

Double Angles

Name: _____

In Exercises 1-10 simplify the given expression.

1. $2 \sin 10^0 \cos 10^0$
2. $\cos^2 15^0 - \sin^2 15^0$
3. $1 - 2 \sin^2 35^0$
4. $2 \cos^2 25^0 - 1$
5. $2 \sin 3x \cos 3x$
6. $\cos^2 5x - \sin^2 5x$
7. $\dfrac{2 \tan 50^0}{1 - \tan^2 50^0}$
8. $\dfrac{2 \tan 40^0}{1 - \tan^2 40^0}$
9. $1 - \sin^2 x$
10. $1 - 2 \sin^2 x$

In Exercises 11-14, A is an acute angle. Use the given information to find sin 2A, cos 2A, and tan 2A.

11. $\sin a = 5/13$
12. $\cos A = 4/5$
13. $\cos A = 24/25$
14. $\tan A = 1/2$

In Exercises 15-19 A is an acute angle.

15. If $\sin A = 3/5$, find sin 2A and sin 4A.
16. If $\cos A = 1/3$, find cos 2A and cos 4A
17. If $\cos A = 1/5$, find cos 2A.
18. If $\cos A = 1/4$, find sin 2A.
19. If $\sin A = 1/2$, find sin 4A.

In Exercises 20-25, evaluate the given expression.

20. $2 \cos^2 \frac{\pi}{8} - 1$
21. $\sin 15^0 \cos 15^0$
22. $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$
23. $\dfrac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$
24. $1 - 2 \sin^2 \frac{\pi}{12}$
25. $4 \sin \frac{\pi}{8} \cos \frac{\pi}{8}$

Half-Angle Formulas Name: _____

In 1-3, complete the identity

1. $\sin \frac{1}{2}x =$ _____
2. $\cos \frac{1}{2}x =$ _____
3. $\tan \frac{1}{2}x =$ _____

Express in terms of cos x:

4. $\sin^2 \frac{1}{2}x =$ _____
5. $\cos^2 \frac{1}{2}x =$ _____
6. $\tan^2 \frac{1}{2}x =$ _____
7. cos x = 1/9. Find $\sin \frac{1}{2}z$. _____
8. cox x = 1/8. Finc cos x/s. _____
9. cos x = 3/5. Find $\tan \frac{1}{2}x$. _____
10. cos x = 0.8. Find $\sin^2 \frac{1}{2}x$. _____

State the quandrant in which $\frac{1}{2}A$ lies if A is an angle in:

11. Quadrant I _____ 12. Quadrant II _____ 13. Quadrant III _____
14. Quadrant IV _____

Multiple Choice:

15. If A is a positive angle in Quandrant III, then

    a. both sin A/2 amd cps A/2 are positive.
    b. sin A/2 is positive and cos A/2 is negative
    c. sin A/2 is negative and cos A/2 is positive
    d. both sin A/2 and cos A/2 are negative

16. The expression $\sqrt{\frac{1 - \cos 80^2}{2}}$ is equivalent to:

    (1) $\cos 40^0$ (2) $\sin 40^0$ (3) $\frac{1}{2} - \cos 40^0$ (4) $\frac{1}{2} \sin 80^0$

17. The espression $\frac{1 - \cos 100^0}{1 + \cos 100^0}$ is equivalent to:

    (1) $\tan 50^0$ (2) $-\cos 100^0$ (3) $\sqrt{\cos 50^0}$ (4) $\tan^2 50^0$

18. The expression $2 \sin^2 \frac{1}{2}\theta$ is equivalent to:

    (1) $1 - \cos \theta$ (2) $1 + \cos \theta$ (3) $\sin^2 \theta$ (4) $2 - 2 \cos \theta$

19. If $\cos \frac{1}{2}\theta = 1/3$, then $\cos \theta$ equals: (1) 2/3 (2) 1/6 (3) -7/9 (4) 7/9

20. Using $\frac{1}{2}(30^0)$, we can fnd the exact value of $\tan 15^0$ (1) $2 + \sqrt{3}$ (2) $2 - \sqrt{3}$
    (3) $\frac{2 + \sqrt{3}}{4}$ (4) $\frac{2 - \sqrt{3}}{4}$

Double and Half-Angle Formulas Name: _____

For Exercise 1-6, if $\sin x = 3/5$, where $0 \leq x \leq \frac{\pi}{2}$, find the value of each expression.

1. $\sin 2x$
2. $\cos 2x$
3. $\tan 2x$
4. $\cos x/2$
5. $\sin x/2$
6. $\tan x/2$

For Exercises 7-12, if $\cos x = -4/5$, where $\pi/2 \leq x \leq \pi$, find the possible values of each expression:

7. $\sin 2x$ _____
8. $\cos 2x$ _____
9. $\tan 2x$ _____
10. $\sin \frac{1}{2}x$ _____
11. $\cos \frac{1}{2}x$ _____
12. $\tan \frac{1}{2}x$ _____

For Exercises 13-18, simplify the given expression.

13. $2 \cos^2 10^0 - 1$
14. $1 - 2 \sin^2 20^0$
15. $\frac{2 \tan 25^0}{1 - \tan^2 25^0}$
16. $2 \sin x/2 \cos x/2$
17. $\frac{4 \tan x}{1 - \tan^2 x}$
18. $\cos^2 4A - \sin^2 4A$
19. Express $\sin 10^0$ in terms of $\cos 20^0$: _____
20. Express $\cos 35^0$ in terms of $\cos 70^0$: _____
21. Express $\tan 40^0$ in terms of $\cos 80^0$: _____

Simplify:

22. $\sqrt{\frac{1 + \cos 70^0}{2}}$ _____
23. $\cos^2 40^0 - \sin^2 40^0$ _____
24. $\sqrt{\frac{1 - \cos 80^0}{2}}$ _____
25. $1 - 2 \sin^2 x/2$ _____

Trig Identity Proofs

Name: _____

1. $\tan x \csc x = \sec x$

2. $\frac{\csc x}{\tan x} = \frac{\cot x}{\cos x}$

3. $\cos^2 x = \csc c \sin x = \sin^2 x$

4. $(1-\cos x)\ (1 + \cos x) = \frac{1}{\csc^2 x}$

5. $\cos \theta \csc \theta = \frac{1}{\tan \theta}$

6. $\frac{1 - \cos A}{\sin A} = \csc A - \text{Cot } A$

7. $\frac{\csc B - 1}{\sec B} = \cot B - \cos B$

8. $\frac{1 + \sec \theta}{\csc \theta} = \sin \theta + \tan \theta$

9. $\sin^2 \theta + \sin^2 \theta \tan^2 \theta = \tan^2 \theta$

10. $\sin 2B = \frac{2 \tan B}{1 + \tan^2 B}$

11. $\frac{2 \sin^2 A}{\sin 2 A} + \cot A = \sec A \csc A$

12. $\tan 2\theta = \frac{2 \tan \theta}{\sec^2 \theta - 2 \tan^2 \theta}$

Trig Identity Proofs

Name: _____

1. $\cos B \csc B = \cot B$

2. $\dfrac{\sin \alpha \csc \alpha}{\cos \alpha} = \sec \alpha$

3. $\tan^2 \gamma + \sin^2 \gamma + \cos^2 \gamma = \sec^2 \gamma$

4. $(1 - \sin^2 \theta)(\sec^2 \theta - 1) = \sin^2 \theta$

5. $\dfrac{1 + \sec A}{\text{tam } A} = \cot A + \csc A$

6. $\dfrac{\sec A + \text{scs } A}{\tan A + \cot A} = \sin A + \text{cas } A$

7. $\dfrac{\sin 2\theta}{1 - \cos^2 \theta + \sin^2 \theta} = \cot \theta$

8. $\sec 2x = \dfrac{1 + \tan^2 x}{1 - \tan^2 x}$

9. $\cos 2A = \dfrac{s - \sec^2 A}{\sec^2 A}$

10. $(\sin x - \cos x)^2 = 1 - \sin 2x$

11. $\dfrac{\cos B}{\sin B} + \dfrac{\sin B \tan B}{\sec B} = \csc B$

12. $\cot^2 \theta = \csc^2 \theta - \sin^2 \theta - \cos^2 \theta$

Factoring

Name: _____

Factor Completely:

1. $\tan^2 x - 9$

2. $\sin^2 x + 3 \sin x + 2$

3. $2 \cos^2 x - \cos x$

4. $\cot^2 x - 9 \cot x + 14$

5. $\sin 2x - 2 \cos x$

6. $15 - 8 \tan x + \tan^2 x$

7. $2 \sin x \cos^2 x - 2 \sin x \cos x - 12 \sin x$

8. $3 \csc^2 x - 7 \csc x + 2$

9. $8 \sin x - 500 s^2 x + 1$

10. $\sec^4 x - 16$

11. $18 \sin^2 x - *8 \sin^2 x + 8 \cos^2 x)$

12. $(\csc^2 x - 1)^4 - 13 \cot^2 x + 36$

Solving Trig Equations Name: _____

1. What value of x between 0^0 and 90^0 satisfies the equation $4 \sin^2 x - 3 = 0$?

2. Find the positive acute angle which satisfies the equation $2 \sin^2 x - 1 = 0$.

3. Find the positive acute angle that satisfies the equation $3 \tan^2 x - 1 = 0$.

4. Solve the following equation for all values of x between 0^0 and $360^)$.
$\cot x \tan 2x = 3$.

Find the positive acute angle x that satisfies the following equations:

5. $2 \cos^2 x - \cos x = 0$

6. $2 \cos^2 x - \sqrt{3} \cos x = 0$

7. $2 \sin^2 x = \sin x$

8. $\cot^2 x - \cot x = 0$

9. $2 \cos^2 x - \sqrt{2} \cos x = 0$

Find, to the nearest degree, the positive acute angle or angles that satisfy the following equations:

10. $\tan^2 x - 4 \tan x + 3 = 0$

11. $6 \sin^2 x - 5 \sin x + 1 = 0$

12. $\tan^2 x - 2 \tan x = 3$

13. $5 \sin^2 x + 1 = 6 \sin x$

Find all angles greater than $0^{'}$ and less than 360^0 that satisfy the following equations: (Express approximate values to the nearest degree.)

14. $6 \cos^2 x + 5 \sin x - 2 = 0$

15. $7 \cos^2 x - 4 \sin x = 4$

16. $2 \sec^2 y + \tan y - 3 = 0$

Solving Trig Equations Name: _____

Find, the the nearest degree, the positive acute angle x which satisfies the following equations:

1. $\tan^2 x = 9$

2. $25 \sin^2 x = 9$

3. $3 \tan^2 x = 12$

4. $9 \cos^2 x = 4$

5. Solve for all values of x between 0^0 and 360^0: $2 \sin^1 x - \cos 2x = 0$

6. Find, correct to the nearest minute, the value of x between 0^0 and 60^0 that satisfies the equation $8 \cos 2x - 4(1 - 2 \sin^2 x) = 1$.

7. Find the positive values of x less than 360^0 that satisfy the equation and check two values: $3 + 2 (\cos^2 x - 1) = \cos x + 1$.

8. Find the smallest positive angle greater than 0^0 which satisfies the equation $\sin^2 x - \sin x = 0$.

9. Find the value of x greater than 0^0 and less than 90^0 which satisfies the equation $2 \sin^2 x - \sqrt{3} \sin x = 0$.

Find, to the nearest degree, all angles greater than 0^0 but less than 360^0 that satisfy the following equations:

10. $\sin^2 x - 3 \sin x + 2 = 0$

11. $\tan^2 x = 14 + 5 \tan x$

12. $6 \tan B = 2 - 5 \tan^2 B$

Find, to the nearest degree, all positive angles less than 360^0 that satisfy the following equations:

13. $2 \cos x + 3 = 2 \sec x$

14. $2 \tan x - 1 = \cot x$

15. Find, to the nearest degree, all values of x between o^0 and 360^0 which satisfy the equation $3 \cos 2x - 7 \cos x = 0$.

16. Find, to the nearest degree, all values of θ greater than 0^0 and less than 360^0 that satisfy the equation $3 \cos 2\theta + 8 \sin\theta + 5 = 0$.

Law of Sine and cosine Applications Name:_____

Solve each problem using logs:

1. Two forces of 30 pounds and 40 pounds act upon a body, forming an acute angle with each other. The angle between the resultant and the 30-pound force is 35^0 10'. Find, to the nearest ten minutes, the angle between the two given forces.

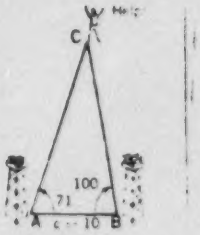
2. A firetower at point A is 10 km north of a firetower at B. A fire at point F is observed from both towers. If the measure of $\angle FAB$ is 68^0 and the measure of $\angle ABF$ is 48^0, find AF.

3. From lighthouses P and Q, 8 km apart, a sinking ship S is sighted. If the measure of $\angle SPQ$ is 32^0 and the measure of $\angle SQP$ is 75^0, find the distance from S to the nearer lighthouse.

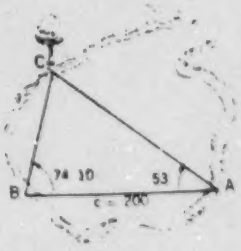
4. Two forces act on a body, making angles of $15^0 40'$ and $37^0 30'$ with the resultant. If the larger force is 42 pounds, what is the magnitude of the resultant to the nearest pound?

5. Two light houses at points A and B on the coast of Maine are 40 km apart. Each has visual contact with a freighter at point C. If $\angle CAB$ measures 20^0 30' and $\angle CBA$ measures 115^0, how far is the freighter from A/ From B?

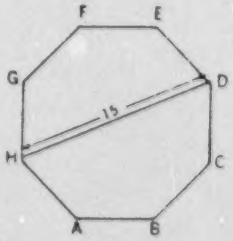
6. Two ranger stations located 10 km apart receive a distress call from a camper. Electronic equipment allows them to determine that the camper is at an angle of 71^0 from the first station and 100^0 from the second, each angle having as one side the line of the stations. Which station is closer to the camper? How far away is it?



7. To find the height of a tree standing at point C across the Huron River from point A, a base line 200 meters long is established on one side of the river. The measure of $\angle BAC$ was found to be 53^0 and that of $\angle CBA$ was $74^0 10'$. The angle of elevation of the top of the tree from A measures 12^0. What is the height of the tree?



8. What is the length of a side of a regular octagon if a diagonal is 15 meters long? See the figure at the right.



Law and Sine and Cosine Applications

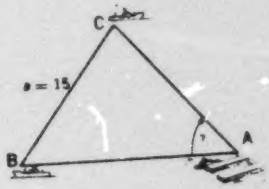
9. A tower 80 meters high stands on a cliff. From the top and botton of the tower, the angles of depression to a ship are 18^0 and 14^0 respectively.

   a. What is the distance of the ship from the foot of the tower?

   b. What is the distance of the ship from the foot of the cliff?

   c. How high is the cliff?

10. Two steamships leave the same port simultaneously, one traveling 10 knots and the other 8 knots. At the end of two hours they are 15 nautical miles apart. What was the measure of the angle between their courses? (Express the measure of angle to the nearest 10'. 1 knot = 1 nautical mile per hr.



11. A woman hikes 6 km due north, then turns to the right and hikes 9 km. If the angle determined by her initial course and her final position measures 40^0, through how many degrees did she turn when she turned to the right? If the angle determined by her initial course and her final position is 60^0, through how many degrees did she turn when she turned to the right?

12. Two jet aircraft leave an airport at the same time. The course of the first is 160^0 east of north while the course of the second is 70^0 west of north. If the first travels 500 mph and the second 600 mph, what is the distance between them at the end of 3 hours?

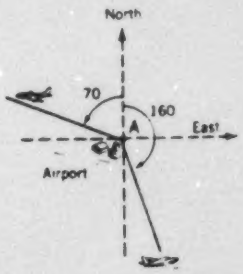


Table Use and Interpolation Name: _____

1. Find the value of sin 27^0 14' to four decimal places. 1. _____

2. Find the value of sin 65^0 23' to four decimal places. 2. _____

3. Find the value of cos 27^0 26' to four decimal places. 3. _____

4. Find the value of tan 73^0 15' to four decimal places. 4. _____

5. Find the vlaue of tan 37^0 17' to four decimal places. 5. _____

6. If sin θ = 0.5319, find the value of θ to the nearest minute. 6. _____

7. If tan A = 0.4750, find the value of A to the nearest minute. 7. _____

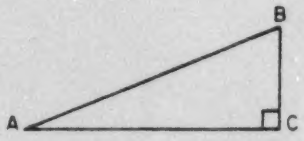
8. Determine all value of θ between 0^0 and 360^0 such that sin θ = 0.1736. 8. _____

9. Determine all value of θ between 0^0 and 360^0 such that sin θ = 0.4067. 9. _____

10. Determine all values of θ between 0^0 and 360^0 such that cos θ = .9850. 10. _____

11. Determine all values of θ between 0^0 and 360^0 such that tan θ = .4023. 11. _____

12. In the accompanying diagram of right triangle ABC, AB = 5000 and m$\angle$C = 90. If the measure of angle A is 20^0 40', find AC to the nearest unit. 12. _____



Radian and Degree Measures Name: _____

1. Express 72^0 in radian measure. 1. _____

2. Express $\frac{7\pi}{10}$ radians in degree measure. 2. _____

3. Express 105^0 in radian measure. 3. _____

4. Express in radian measure an angle of 240^0. 4. _____

5. Express in degrees an angle whose radian measure is $\frac{7\pi}{12}$ 5. _____

6. Express 390^0 in radian measure. 6. _____

7. Express $\frac{4\pi}{3}$ radians in degrees. 7. _____

8. Express 100^0 in radian measure. 8. _____

9. Express $\frac{7\pi}{10}$ radians in degree measure. 9. _____

10. Express in radian measure an angle of 540^0. 10. _____

11. Express in degree measure an angle of $\frac{7\pi}{3}$ 11. _____

12. Express in radian measure an angle of 130^0. 12. _____

13. Express 144^0 in radian measure. 13. _____

14. Find the measure, in radians, of an angle whose degree measure is 120^0. 14. _____

15. One radian is approximately equal to: 15. _____

(1) 57^0 (3) π

(2) 45^0 (4) $\frac{\pi}{2}$

Quadrant Placement Name: _____

Multiple Choice:

1. If $\sin\theta$ and $\tan\theta$ have opposite signs, in which quadrants may angle θ lie? 1. _____
   (1) I and II (3) I and III
   (2) II and III (4) II and IV

2. If $\sin\theta = \frac{1-\sqrt{17}}{4}$, then angle θ lies in which quadrants? 2. _____
   (1) I and II, only (3) III and IV, only
   (2) II and IV, only (4) I, II, III, and IV

3. If $\cos x > 0$, then which must always be true? 3. _____
   (1) $\sin x > 0$ (3) $\sec x > 0$
   (2) $\tan x > 0$ (4) $\csc x > 0$

4. If $\tan x = -\frac{\sqrt{3}}{3}$, in which quadrant(s) may angle x terminate? 4. _____
   (1) II and III (3) III, only
   (2) II, only (4) II and IV

5. If $\cos x = -4/5$, and $\tan x > 0$, then $\angle x$ terminates in Quadrant 5. _____
   (1) I (3) III
   (2) II (4) IV

6. If $\sin\theta < 0$ and $\tan\theta < 0$, the measure of θ may be 6. _____
   (1) -80^0 (3) 120^0
   (2) -100^0 (4) 220^0

7. If $\cos x = -4/5$ and $\tan x > 0$, then $\angle x$ terminates in Quadrant 7. _____
   (1) I (3) III
   (2) II (4) IV

8. If $\sin x \cos x > 0$ and $\csc x < 0$, then angle x in in quadrant 8. _____
   (1) I (3) III
   (2) II (4) IV

9. If $\sin A < 0$, in which quadrants may angle A lie? 9. _____
   (1) I, II (3) II, IV
   (2) II, III (4) III, IV

Quadrant Placement

Short Answer:

10. In which quadrant are tangent and cosecant both negative? 10. _____

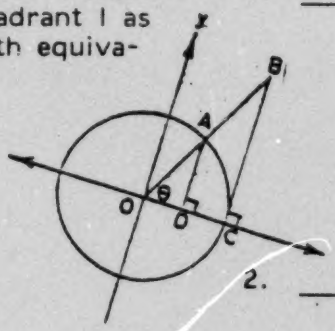
11. If tan $\theta < 0$ and sin $\theta = 3/5$, then angle θ is in quadrant_____. 11. _____

12. If sin $x = -2/5$ and cos $x > 0$, in which does angle x terminate? 12. _____

Angle Reduction Name: _____

Multiple Choice:

1. In the accompanying figure, a unit circle is drawn with radius OA = 1. Angle O is in quadrant I as shown. Which line segment has a length equivalent to cos O? 1. _____



(1) $\overline{OB}$ (3) $\overline{OD}$

(2) $\overline{OA}$ (4) $\overline{OC}$

2. If $f(x) = \cos x + \tan x/3$, then $f(\pi)$ is 2. _____

(1) $\frac{\sqrt{3}+3}{3}$ (3) $\sqrt{3}+1$

(2) $\frac{\sqrt{3}-3}{3}$ (4) $\sqrt{3}-1$

3. If $\sin x = \sqrt{3}\cos x$, then x equals 3. _____

(1) 30^0 (3) 60^0

(2) 45^0 (4) 90^0

4. The expression $\tan(-230^0)$ is equivalent to 4. _____

(1) $\tan 40^0$ (3) $\tan 50^0$

(2) $-\tan 40^0$ (4) $-\tan 50^0$

Short Answer:

5. If a function f is defined by $f(x) = \sin(x/2)$ find the numerical value of $f(\pi)$. 5. _____

6. Express $\sin 230^0$ as a function of a positive acute angle. 6. _____

7. Express $\sin(-170^0)$ as a function of a positive acute angle. 7. _____

8. Express $\cos(-175^0)$ as a function of a positive acute angle. 8. _____

9. Express $\cos(-100^0)$ as a function of a positive acute angle. 9. _____

Angle Reduction

10. Express $\cos 200^0$ as a function of a positive acute angle. 10. _____

11. Express $\tan (-150^0)$ as a function of a positive acute angle. 11. _____

12. Find the value of $\sin 210^0$. 12. _____

13. Find the value of $\tan \frac{3\pi}{4}$. 13. _____

14. Find the value of $\cos \frac{5\pi}{3}$. 14. _____

15. Find the value of $\tan 315^0$. 15. _____

Angle Reduction Name: _____

Multiple Choice:

1. Which number is not an element of the range of the function defined by $y = \cos x$? 1. _____

(1) 1 (3) 0
(2) 2 (4) $\frac{1}{2}$

2. The value of $\tan \frac{5\pi}{4}$ is 2. _____

(1) 1 (3) $\sqrt{3}$
(2) -1 (4) $\frac{\sqrt{3}}{3}$

3. If $f(x) = 2 \cos x + \sin 2x$, the numerial value of $f\left(\frac{\pi}{2}\right)$ is 3. _____

(1) -1 (3) 0
(2) 2 (4) -2

4. If $f(x) = \cos x + \tan x/3$, then $f(\pi)$ is 4. _____

(1) $\frac{\sqrt{3} + 3}{3}$ (3) $\sqrt{3} + 1$
(2) $\frac{\sqrt{3} - 3}{3}$ (4) $\sqrt{3} - 1$

5. Which is equivalent to $\sin 73^0$? 5. _____

(1) $\cos 17^0$ (3) $\sin 17^0$
(2) $\cos 73^0$ (4) $-\sin 73^0$

6. The value of $\tan 315^0$ is equivalent to 6. _____

(1) $\sin 90^0$ (3) $\sin 270^0$
(2) $\cos 90^0$ (4) $\cos 270^0$

Short Answer:

7. If a function f is defined by $f(x) = \sin 2x$, find the numberical value of $f(\pi/4)$. 7. _____

8. Express $\sin 230^0$ as a function of a positive acute angle. 8. _____

Angle Reduction

9. Express cos (-230^0) as a function of a positive acute angle. 9. _____

10. Express tan (-155^0) as a function of a positive acute angle. 10. _____

11. Express tan (-230^0) as a function of a positive acute angle. 11. _____

12. Express cos 128^0 as a function of a positive acute angle. 12. _____

13. Express cot 212^0 as a function of a positive acute angle. 12. _____

14. What is the value of sin (-330^0)? 14. _____

15. Evalutate cos $\frac{4\pi}{3}$. 15. _____

16. Find the value of sin 750^0? 16. _____

Inverse Functions

Name: _____

1. Find cos (arc tan $\sqrt{3}$).
2. Find tan $\left(\text{Arc sin } \frac{\sqrt{2}}{2}\right)$
3. Find cos (Arc tan 1).
4. Find tan (Arc sin 3/5).
5. Find the value fo cot (Arc tan 8/5).
6. Find the value of sin (Arc cos 15/17).
7. If Θ = Arc cos $\left(\frac{\sqrt{3}}{2}\right)$, what is the measure of angle Θ?
8. Find tan $\left(\text{Arc sin } \frac{\sqrt{3}}{2}\right)$.
9. Find the positive value of tan (Arc sin 12/13).
10. Find the positive value of tan (arc cos 3/5).
11. Find the positive value of tan $\left(\text{Arc cos } \frac{\sqrt{2}}{2}\right)$.
12. If Θ = Arc tan (1), what is the measure of acute angle Θ?
13. If A = Arc tan 3/4. then what is the value of tan A/2?

    (1) $\sqrt{1/7}$ (3) 1/3
    (2) 1/2 (4) 1/4

14. If Θ = Arc cos $(\frac{1}{2})$, the value of sin Θ is

    (1) 1/2 (3) $\frac{\sqrt{3}}{2}$
    (2) -1/2 (4) $-\frac{\sqrt{3}}{2}$

15. If x = arc cos $(-\frac{1}{2})$, then x is equal to:
    (1) 30^0 (3) 120^0
    (2) 60^0 (4) 150^0

Amplitude and Period Name: _____

1. What is the period of the graph of $y = \frac{1}{2} \cos 2x$? 1. _____

2. What is the amplitude of the graph of $y = 2 \sin \frac{1}{2}$? 2. _____

3. What is the amplitude of the graph of $y = 3 \cos 2x$? 3. _____

4. What is the amplitude of the graph of y = cox 2x? 4. _____

5. What is the period of $y = 3 \sin \frac{1}{2}$? 5. _____

Multiple Choice:

6. Which is the maximum value of the function $y = 2 \sin 3x$? 6. _____
   (1) 1 (3) 3
   (2) 2 (4) 4

7. The period of the curve $y = \frac{1}{2} \cos 2x$ is 7. _____
   (1) 1/2 (3)
   (2) 2 (40 4

8. The maximum value of $f(\theta) = 2 \sin 3\theta$ is 8. _____
   (1) 1 (3) 3
   (2) 2 (4)

9. The period of $y = 3 \sin 2x$ is: 9. _____
   (1) (3) 3
   (2) 2 (4) 4

10. The amplitude of $y = 3 \sin 2x$ is: 10. _____
   (1) 1 (3) 3
   (2) 2 (4) 4

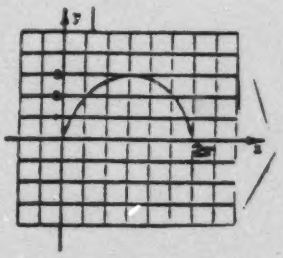
11. The maximum value of the function $y = 3 \sin 2x$ is: 11. _____
   (1) (3) 3
   (2) 2 (4) 2

12. What is the period of the graph of $y = 3 \sin 2x$? 12. _____
   (1) π (3) 3
   (2) 4π (4) 2

Graphing Circular Functions Name: _____

Multiple Choice:

1. What is the equation of the function sketched in accompanying diagram? 1. _____



2. The equation $y = \tan x$ is graphed in the interval $0 \leq x < \pi/2$ and is reflected over the x-axis. On this reflection, point $(\pi/4, y)$ has which value of Y? 2. _____

3. As angle x increases from 90^0 to 180^0, the value of sin x 3. _____
   (1) increases from -1 to 0
   (2) increases of 0 to 1
   (3) decreases from 0 to -1
   (4) decreases from 1 to 0

USE GRAPH PAPER:

4. a. On the same set of axes, sketch and lable the graphs of $y = \tan x$ and $y = 2 \cos x$ for the values of x in the interval $o \leq x < 2\pi$.
   b. State the number of values of x in the interval $o \leq x < 2\pi$ which satisfy the equation $\tan x = 2 \cos x$.

5. a. On the same set of axes, sketch the graphs of $y = \cos 3x$ and $y = \frac{1}{2} \sin x$ for the values of x in the interval $o \leq x < 2\pi$.
   b. How many values of x in the interval $0 \leq x \leq 2\pi$ satisfy the equation $\cos 2x = \frac{1}{2} \sin x$?

6. a. On the same set of axes sketch and label the graphs of $y = \frac{1}{2}\cos x$ and $y = 2x$ for values of x in the interval $-\pi < x < \pi$.
   b. Find the graphs made in part a, determine the number of values of x in the interval $-\pi < x < \pi$ which satisfy the equation $\frac{1}{2} \cos x = \sin 2x$.

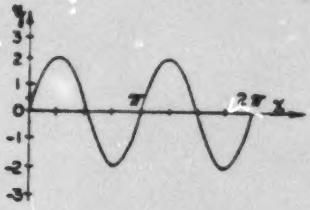
Graphing Circular Functions Name: _____

Multiple Choice:

1. Which is the equation of the function sketched in the accompanying diagram? 1. _____

(1) $y = 2 \sin \frac{1}{2}x$ (3) $y = 2 \sin 2x$

(2) $y = 1 \cos 2x$ (4) $y = 2 \cos 2x$



2. As θ increases form $\pi/2$ to π, which statement is true? 2. _____

(1) sin O decreases from 0 to -1

(2) cos O decreases from 0 to -1

(3) cos O increases from -1 to 0

(4) sin O increases from -1 to 0

3. As angle x increases from 180^0 to 270^0, the value of cos x will 3. _____

(1) increase from 0 to 1

(2) increase from -1 to 0

(3) decrease from 0 to -1

(4) decrease from 1 to 0

ON GRAPH PAPER

4. a. On the same set of axes, sketch the graphs of $y = 3 \cos x$ and $y = \sin (x/2)$ for the values of x in the interval $0 \leq x \leq 2\pi$.

b. State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $\sin (x/2) = 3 \cos x$.

5. a. On the same set of axes, sketch the graphs of $y = \tan x$ and $y = 2 \cos x$ over the interval $0 \leq x \leq 2\pi$.

b. In the interval $o \leq x \leq 2\pi$, for how many values does $\tan 2 = 2 \cos x$?

6. a. On the same set of axes, sketch the graphs of $y = 2 \sin x$ and $y = \cos \frac{1}{2}x$ as x varies from 0 to 2π radians.

b. State the number of values of x in the interval $o \leq x \leq 2\pi$ that satisfy the equation $2 \sin x = \cos \frac{1}{2}x$.

Graphing Circular Functions Name: _____

USE GRAPH PAPER, SOLVE:

1. a. On the same set of axes, sketch the graphs of $y = \tan x$ and $y = \frac{1}{2}\cos x$, as x varies from 0 to 2π radians.
   b. State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $\tan x = \frac{1}{2}\cos x$.

2. a. On the same set of axes, sketch and label the graphs of $y = 2 \sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians.
   b. How many values of x in the interval $0 \leq x \leq 2\pi$ satisfy the equation $2 \sin x = \cos 2x$?

3. a. On the same set of axes, sketch the graphs of $y = \cos \frac{1}{2}x$ and $y = \tan x$ for the values of x in the interval $0 \leq x \leq 2\pi$.
   b. How many values of x in the interval $0 \leq x \leq 2\pi$ satisfy the equation $\cos \frac{1}{2}x = \tan x$?

4. a. On the same set of axes, sketch the graphs of $y = \frac{1}{2} \cos 2x$ and $y = \sin x$ for the values of x in the interval $0 \leq x \leq 2\pi$.
   b. How many values of x in the interval $0 \leq x \leq 2\pi$ satisfy the equation $\frac{1}{2} \cos 2x = \sin x$.

5. a. On the same set of axes, sketch the graphs of $y = 2 \cos x$ and $y = \sin 2x$ for the values of x in the interval $0 \leq x \leq 2\pi$.
   b. How many values of x in the interval $0 \leq x \leq 2\pi$ satisfy the equation $2 \cos x = \sin 2x$?

6. a. On the same set of axes, sketch the graphs of $y = \cos 2x$ and $y = \sin x$ for the values of x in the interval $0 \leq x \leq 2\pi$.
   b. State the number of values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $\cos 2x = \sin x$.

7. a. On the sme set of axes, sketch the graphs of $y = \frac{1}{2}\sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians.
   b. From the graphs made in part a, determine the number of values of x between 0 and 2π radians which satisfy the quation $\frac{1}{2}\sin x = \cos 2x$.

Sum and Difference Formulas Name: _____

1. The expression $\cos(90^0 + \theta)$ equals 1. _____
   (1) $\cos\theta$ (3) $\sin\theta$
   (2) $-\cos\theta$ (4) $-\sin\theta$

2. The expression $\sin(\theta + 270^0)$ equals 2. _____
   (1) $\cos\theta$ (3) $-\cos\theta$
   (2) $2\cos\theta$ (4) $-\sin\theta$

3. The expression $\cos(2\pi - x)$ is equivalent to 3. _____
   (1) $-\cos x$ (3) $-\sin x$
   (2) $\cos x$ (4) $\sin x$

4. The expression $\cos(x + 90^0)$ is equivalent to 4. _____
   (1) $-\sin x$ (3) $\cos x$
   (2) $\sin x$ (4) $\cos x + \sin x$

5. If x and y are positive acute angles, and $\sin x = 3/5$ and $\sin y = 1/2$, then $\sin(x+y)$ is equal to 5. _____
   (1) $\frac{3\sqrt{3} - 4}{10}$ (3) $\frac{12}{25} + \frac{\sqrt{3}}{4}$
   (2) $\frac{3\sqrt{3} + 4}{10}$ (4) $\frac{12}{25} - \frac{\sqrt{3}}{4}$

6. If $\sin x = 3/5$, and x is a positive acute angle, find $\cos(x + \pi/2)$. 6. _____

7. If $\tan x = 1/2$ and $\tan y = 1$, the value of $\tan(x+y)$ is 7. _____
   (1) 1/2 (3) 3
   (2) 3/4 (4) 3/2

8. The expression $\cos(90^0 - x)$ is equivalent to 8. _____
   (1) $-\cos x$ (3) $-\sin x$
   (2) $\cos x$ (4) $\sin x$

9. If $\sin(A - 30)^0 = \cos 60^0$, the number of degrees in the measure of angle A is 9. _____
   (1) 30 (3) 90
   (2) 60 (4) 120

10. The expression $(\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0)$ is equal to
(1) 1 (3) $\frac{\sqrt{3}}{2}$
(2) 1/2 (4) 0

10. _____

11. If $\angle x$ is an acute angle and $\sin x = 3/5$, find the value of $\sin (180^0 - x)$.

11. _____

12. The value of $(\sin 13^0)(\cos 17^0) + (\cos 13^0)(\sin 170)$ equals
(1) 1 (3) $\frac{\sqrt{3}}{2}$
(2) 1/2 (4) 0

12. _____

13. If $\tan (A - 30) = \cot A$, the number of degrees in the measure of angle A is
(1) 30 (3) 60
(2) 45 (4) 90

13. _____

14. If x and y are positive acute angles, and $\sin x = 3/5$ and $\sin y = 1/2$, the $\cos (x + y)$ is equal to
(1) $\frac{4\sqrt{3} + 3}{10}$ (3) $\frac{4}{5} + \frac{\sqrt{3}}{2}$
(2) $\frac{4\sqrt{3} - 3}{10}$ (4) $\frac{4}{5} - \frac{\sqrt{3}}{2}$

14. _____

HALVING FORMULAS: Name:_____

1. Find the positive value of sin ½x if cos x = 0.02.

2. If cos x = 3/5 and x is a positive acute angle, find tan ½ x.

3. If x is a positive acute angle and cos x = 7/18, find cos ½x.

4. If cos x = 3/5, what is the positive value of sin ½x?

DOUBLING FORMULAS

1. If sin x = 5/6, what is the value of cos 2x?

2. If sin x = 3/5, what is the value of cos 2x?

3. If angle x is an acute angle and sin x = 5/13, find the value of sin 2x.

4. If x is a positive acute angle and cos x = 5/13, find sin 2x.

Trig Identity Proofs Name: _____

1. The expression $\sin 2\theta \sec \theta$ is equivalent to 1. _____
   (1) $\tan \theta$ (3) $2 \cos \theta$
   (2) $\tan 2\theta$ (4) $2 \sin \theta$

2. The expression $\csc y + 1$ is equivalent to 2. _____
   (1) $\dfrac{\cot y}{\csc y - 1}$ (3) $\cot y$
   (2) $\dfrac{\sin y + 1}{\sin y}$ (4) $\dfrac{1 + \cos y}{\cos y}$

3. The expression $(\cot \theta)(\sec \theta)$ is equivalent to 3. _____
   (1) $\csc \theta$ (3) $\cos \theta$
   (2) $\sin \theta$ (4) $\tan \theta$

4. Express as a single function: $\dfrac{\dfrac{1}{\cos \theta} - \cos \theta}{\dfrac{\sin \theta}{\cos \theta}}$ 4. _____

5. Prove the identity $\dfrac{\sin x - \cos y}{\sin x + \cos y} = \dfrac{\sec y - \csc x}{\sec y + \csc x}$. 5. _____

6. Prove the identity $\dfrac{\sin \theta \tan \theta + \cos \theta}{\cos \theta} = \sec^2 \theta$. 6. _____

7. For all values of x for which the expressions are defined, prove the identity: 7. _____
   $\dfrac{\tan \theta \csc^2 \theta}{1 + \tan^2 \theta} = \cot \theta$

8. For all values of θ for which the expressions are defined, prove that the following is an identity: 8. _____
   $\dfrac{\cos \theta + \cot \theta}{\cos \theta \cot \theta} = \tan \theta + \sec \theta$

9. For all values for which the expressions are defined prove the following is an identity: 9. _____
   $\sin 2x = \dfrac{2 \tan x}{1 + \tan^2 x}$

10. a. Find all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $2 \cos^3 \theta + \cos^2 \theta - \cos \theta = 0$. 10a. _____
    b. For all values of θ for which the expressions are defined, prove the following identity: b. _____

    $2 \csc^2 \theta = \dfrac{1}{1 + \cos \theta} + \dfrac{1}{1 - \cos \theta}$

11. The expression $(\sin x - \cos x)^2$ is equivalent to 11. _____
    (1) 1 (3) $1 - \sin 2x$
    (2) $-\cos 2x$ (4) $1 - \cos 2x$

Trig Identity Proofs Name: _____

1. The expression $\sin 2\theta \csc \theta$ is equivalent to 1. _____
   (1) $\tan \theta$ (3) $2 \cos \theta$
   (2) $\tan 2\theta$ (4) $2 \sin \theta$

2. The expression $\cos 2\theta \sec 2\theta$ is equivalent to 2. _____
   (1) 1 (3) $\cos^2 2\theta$
   (2) 2 (4) $\sin^2 2\theta$

3. The expression $\frac{1}{\sin^2 A} - 1$ is equivalent to 3. _____
   (1) $\cot^2 A$ (3) $\sec^2 A - 1$
   (2) $\cos^2 A$ (4) $\frac{\sin^2 A - 1}{\sin^2 A}$

4. The expression $(\tan \theta)(\csc \theta)$ is equivalent to 4. _____
   (1) $\sin \theta$ (3) $\operatorname{scs} \theta$
   (2) $\cos \theta$ (4) $\sec \theta$

5. Prove the identity: 5. _____
   $$\frac{\cos 2\theta}{\sin \theta} - \sin \theta = \frac{\cot \theta}{\sec \theta}$$

6. Prove the identity: 6. _____
   $$(1 + \operatorname{scs}\theta)(1 - \sin \theta) = \cot \theta \cos \theta$$

7. Prove the identity: 7. _____
   $$\frac{\cos \theta}{\sin \theta \tan \theta + \cos \theta} = \frac{1}{\sec^2 \theta}$$

8. For all values of x for which the expressions are defined, prove the identity: 8. _____
   $$\frac{\tan \theta \csc^2 \theta}{1 + \tan^2 \theta} = \cot \theta$$

9. For all values of θ for which the expressions are defined, prove the identity: 9. _____
   $$\tan \theta + \cot \theta = \sec \theta \csc \theta$$

10. a. For all values of x for which the expression is defined, show that the following equality is an identity: 10a. _____
    $$2 - \sec^2 x = (\cos 2x)(\sec^2 x)$$
    b. Solve for θ in the interval $0^0 \leq \theta \leq 360^0$ b. _____
    $$\cos 2\theta = -\sin \theta$$

11. If $\sin x \tan x > 0$ and $\tan x > 0$, which is true? 11. _____
    (1) $\cos x$ 0 (3) $\sec x$ 0
    (2) $\cos x$ 0 (4) $\cot x$ 0

Trig Equations Name: _____

Solve:

1. Solve for θ: $4 \sin^3 \theta - \sin \theta = 0$.

2. Find the measure of the positive acute angle which satisfies the equation $4 \sin^2 \theta - 1 = 0$.

3. What value of x satisfies th equation $\sin x \cos x - 2 \cos x = 0$ for $0^0 < x < 180^0$?

4. Find the value of x in the interval $90^0 < x < 180^0$ which satisfies the equation $\cos x - 2 \cos x \sin x = 0$

5. Find the value(s) of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $\cos^2 \theta - \cos \theta - 2 = 0$.

6. Find the measure of the acute angle for which $2 \cos^2 \theta - \cos \theta = 0$.

7. Find the measure of the smallest acute angle for which $2 \sin^2 \theta - 3 \sin \theta + 1 = 0$.

8. If $0^0 \leq \theta \leq 360^0$, solve for $\sin \theta$: $\sin^2 \theta - \sin \theta - 2 = 0$.

9. In the interval $0^0 \leq \theta \leq 360^0$, how many values of θ satisfy $\tan^2 \theta - 1 = 0$?
(1) 1 (3) 3
(2) 2 (4) 4

10. In the interval $0^0 \leq \theta \leq 360^0$, how many values of θ satisfy the equation $3 \sin^2 \theta + \sin \theta - 2 = 0$?
(1) 1 (3) 3
(2) 2 (4) 4

11. Which is the solution of the equation $\sin x = \cos 2x$?
(1) 15^0 (3) 30^0
(3) 60^0 (4) 45^0

12. How many solutions to the equation $2 \sin^2 x + 3 \sin x + 1 = 0$ are in the interval $0^0 \leq x \leq 90^0$?
(1) 1 (3) 3
(2) 2 (4) 0

Trig Equations - Part II Questions Name: _____

1. Find all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $2\sin^2\theta - 3\sin\theta + 1 = 0$

2. Find, to the nearest degree, all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $\tan\theta - 2\cot\theta = 1$.

3. Find all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $\cos 2\theta + \cos\theta + 1 = 0$.

4. Find all values of x in the interval $0^0 \leq x \leq 360^)$ which satisfy the equation $\tan^2 x - 1 = 0$.

5. Find all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $\tan\theta + \tan\theta\sin\theta = 0$.

6. Find to the nearest degree all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $\sec^2\theta = 4 - 2\tan\theta$.

7. Find all values of θ in the interval $0 \leq \theta \leq 360^0$ which satisfy the equation $\cos 2\theta = \sin\theta$.

Trig Equations - Part II Questions Name: _____

1. Solve the equation $1 + \sin x = 2\cos^2 x$ for a positive acute angle.

2. Find all values of θ in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $2\sin\theta + 1 = \csc\theta$.

3. Find, to the nearest degree, all values of x in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $3\cos 2\theta + \sin\theta - 2 = 0$.

4. Find, to the nearest degree, all values of O in the interval $0^0 \leq \theta \leq 360^0$ which satisfy the equation $7\cos\theta + 1 = 6\sec\theta$.

5. a. Find to the nearest tenth, the value(s) of $\tan\theta$ which satisfy the equation $\tan^2\theta - 3\tan\theta + 1 = 0$.
   b. Using the answer(s) obtained in prt a, find the quadrant(s) in which angle θ may lie.

6. Solve for $\cos\theta$: $\cos\frac{\theta}{2} = \frac{\sqrt{5}}{4}$

Law of Sines Name: _____

1. In triangle ABC, a = 5, sin a = 1/8, and sin B = 1/4. Find the value of b.

2. In triangle ABC, m∠A = 40, m∠B = 50, and side c = 80 centimeters. Find side b to the nearest centimeter.

3. In △RST, r = 3, s = $3\sqrt{2}$, and m∠R = 30. Find m∠S if S in an acute angle.

4. In triangle ABC, sin A = 0.8, sin b = 0.3, and a = 24. Find the length of side b.

5. In triangle ABC, b = 4, c = 3, and sin b = ½. Find the value of sin C.

6. If a = 12, sin A = 2/3, and sin B = 1/2, find b.

7. In △RST, sin R = 0.6, sin S = 0.4m ~~abd~~ ~~eude~~ and side s = 16. Find the length of side r.

8. In triangle ABC, a = 2, b = 10, and sin A = 1/6. Find sin B.

9. If in △ABC, a = $\sqrt{2}$, △A = 30^0, and △B = 45^0, then b is

   (1) 1/2 (3) 3 $\frac{\sqrt{2}}{2}$

   (2) 2 (4) 4

10. In △ABC, m∠A = 30, m∠C = 45, and a = 8 2. What is the value of c?

    (1) 32 (3) $4\sqrt{2}$

    (2) 16 (4) 4

Law of Sines

11. On a level piece of ground, two surveyors establish a baseline running from A to B, a distance of 120 meters. From A and from B, a point C is sighted and $\angle CAB$ and $\angle CBA$ are measured to be $51^0 30'$ and $93^0 10'$ respectively. Find AC to the nearest meter.

12. In trapezoid ABCD, AB and DC are parallel bases and $AD \perp AB$, diagonal AC is drawn, $m\angle BCD = 128$, $m\angle DAC = 42$, and $AD = 26$. Find AB to the nearest integer.

13. From a point on the level ground, a surveyor finds the angle of elevation of the top of a TV tower to be $31^0 40'$. After walking 70 meters in a straight line toward the tower, he finds that the angle of elevation of the top of the tower is $58^0 20'$. Find the height of the tower to the nearest meter.

14. In triangle ABC, $m\angle A = 27^0 10'$, $m\angle B = 33^0 20'$, and side BC = 12 centimeters. Find th length of side $\overline{AB}$ to the nearest centimeter.

Ambiguous Case Name: _____

Indicate whether a solution exists and if so the number of solutions for each set of data. Do not solve.

1. $a = 2, b = 3, c = 6$
2. $C = 17^0, a = 10, c = 1$
3. $B = 71^0, a = 5, c = 275$
4. $A = 20^0, a = 7, b = 20$
5. $A = 41^0, B = 160^0, a = 10$
6. $C = 30^0, b = 10, c = 4$
7. $A = 60^0, b = 2, a = \sqrt{3}$
8. $A = 90^0, a = 20, b = 19$
9. $B = 140^0, a = 3, b = 2$
10. $C = 120^0, b = 14, c = 13$

11. Given: $m\angle A = 52$, side $a = 16$, and side $b = 18$. What is the total number of noncongruent triangles that can be constructed?

12. If $a = 4$, $b = 5$, and $m\angle A = 30$, the number of distinct triangles that may be constructed is

(1) 1 (3) 3
(2) 2 (4) 0

13. If $a = 5$, $b = 7$, and $m\angle A = 30$, how many distinct triangles can be constructed?

(1) 1 (3) 3
(2) 2 (4) 4

14. In $\triangle ABC$, if $m\angle A = 30$, $b = 10$, and $a = 12$, then $\triangle ABC$

(1) must be a right triangle
(2) must be an acute triangle
(3) must be an obtuse triangle
(4) may be either an acute or an obtuse triangle

Law of Cosines Name: _____

1. In $\triangle PQR$, p = 6, 1 = 8, and r = 10. the cosine of the largest angle of the triangle is _____.

2. In $\triangle ABC$, a = 3, b = 4, and $m\angle C = 60$. What is the value of c?

3. In $\triangle ABC$ a = 6, b = 5, and cos C = 3/4. The length of c is _____?

4. In $\triangle ABC$, a = 4, b = 3, and $c = \sqrt{13}$. Find cos C.

5. In triangle ABC, a = 2, b = 3, and c = 4. What is the value of cos C?

6. In triangle ABC, a = 2, b = 3, and c = 4. What is the value of cos A?

7. In $\triangle ABC$, if a = 4, b = 9, and c = 8, then cos A equals _____?

8. If, in $\triangle ABC$, a = 5, b = 6, and c = 8, then cos A is _____?

9. In triangle ABC, $a = \sqrt{3}$, $b = \sqrt{3}$, and $m\angle C = 120$. Find the value of c.

10. In $\triangle ABC$, a = 3, b = 8, and $m\angle C = 60$. Find the length of side C.

Law of Cosines Name: _____

1. In triangle ABC, a = 2, b = 5, and cos C = ½. The length of side c is _____?

2. In $\triangle ABC$, a = 3, b = 4, and c = 5. Find the value of cos C.

3. In triangle ABC, a = 2, b = 4, and $m\angle C = 60$. What is the value of c?

4. If, in $\triangle ABC$, a = 5, b = 6, and c = 8, then cos A is _____?

5. In triangle ABC, a = 12, b = 10, and the measure of angle c is $41^0 30'$.

   a. Determine the length of side c to the <u>nearest integer</u>.

   b. Using your answer to part a, determine the measure of angle A to the <u>nearest degree</u>.

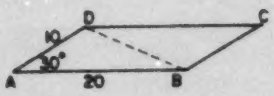
Area Name: _____

1. In triangle ABC, m∠C = 90, side c = 13 centimeters, and side a = 5 centimeters. Find the area of the triangle in square centimeters.

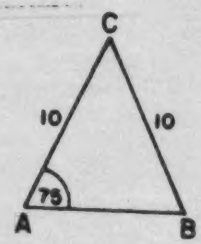
2. Find, to the nearest integer the area of parallelogram ABCD if AB = 10. BC = 8, and m∠A = 50.

3. Find the area of triangle ABC if a = 12, c = 18, and m∠B = 30.

4. In the accompanying diagram, ABCD is a parallelogram with AB = 20, AD = 10, and m∠A = 30. Find the area of parallelogram ABCD.



5. In the accompanying figure of ∠ABC, a = 10, b = 10, and m∠A = 75. Find the area of △ABC.



6. In isosceles △ABC, AC = BC = 10. The measure of angle ACB = 31^0 20'.

   a. Find side ab to the nearest tenth.
   b. Find the area of △ABC to the nearest integer.

7. In rhombus ABCD, the measure of △ABC = 100^0 40' and the length of each side is 5.

   a. What is the length of diagonal $\overline{AC}$ to the nearest tenth?
   b. Find the area of ABCD to the nearest integer.

Area Name: _____

1. a. Two sides of a triangular plot measure 30 meters and 18 meters, respectively. If the angle opposite the 30 meter side measures 58^0, find, to the nearest degree, the measure of the angle opposite the 18 meter side.

   b. Using the answer to part a, find the area of the triangle to the nearest square meter.

2. In triangle ABC, b = 20, c = 12, and $m\angle A = 28^0 50'$.

   a. Find the length of side a to the nearest unit.

   b. Find the area of $\triangle ABC$, to the nearest square unit.

3. a. Two consecutive sides of a parallelogram are 8 centimeters and 10 centimeters, respectively. If the length of the longer diagonal of the parallelogram is 14 centimeters, find the measure of the largest angle of the parallelogram to the nearest degree.

   b. Using your answerto part a, find the area of the parallelogram to the nearest square centimeter.

4. a. In $\triangle ABC$, a = 11, b = 20, and $m\angle B = 42^0\ 30'$. Find the measure of angle A to the nearest ten minutes.

   b. Using the result obtained in part a, find the area of $\triangle ABC$, to the nearest square unit.

FORMULAS

Pythagorean and Quotient Identities

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$\cot^2 A + 1 = \csc^2 A$

$\tan A = \frac{\sin a}{\cos a}$

$\cot A = \frac{\cos A}{\sin A}$

Functions of the Sum of Two Angles

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\cos (A + B) = \cos A \cos B = \sin A \sin B$

$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Functions of the Difference of Two Angles

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Functions of the Double Angle

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Functions of the Half Angle

$\sin \frac{1}{2}A = \pm\sqrt{\frac{1 - \cos A}{2}}$

$\cos \frac{1}{2}A = \pm\sqrt{\frac{1 + \cos A}{2}}$

$\tan \frac{1}{2}A = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of Triangle

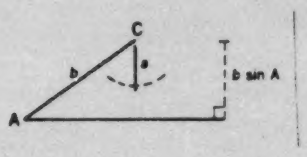
$K = \frac{1}{2}ab \sin C$

Solving Traingle: The Ambiguous Case Name: _____

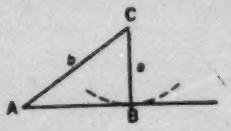
Given two sides and the angle opposite one of them (SSA)

Suppose the data set includes A, b and a. the situations pictured below may result for $A < 90^0$ and for $A \geq 90^0$.

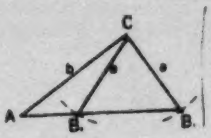
$A < 90^0$



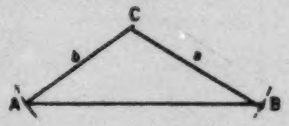
No solution: $a < b \sin A$



One solution: $a = b \sin A$

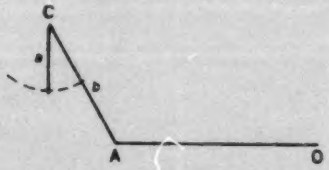


Two solutions $b \sin A < a < b$

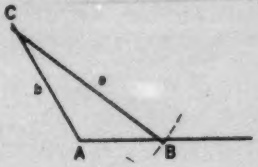


One solution: $a > b$

$A \geq 90^0$

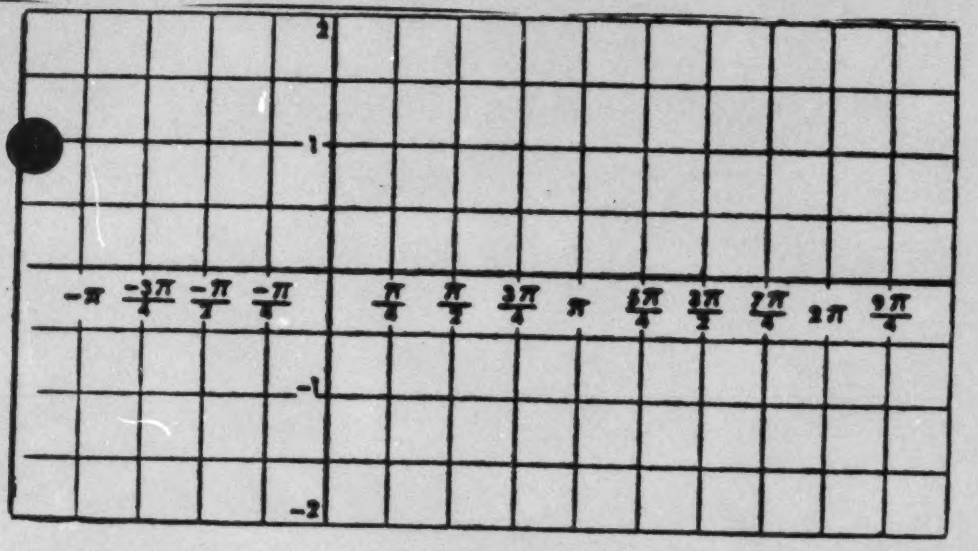


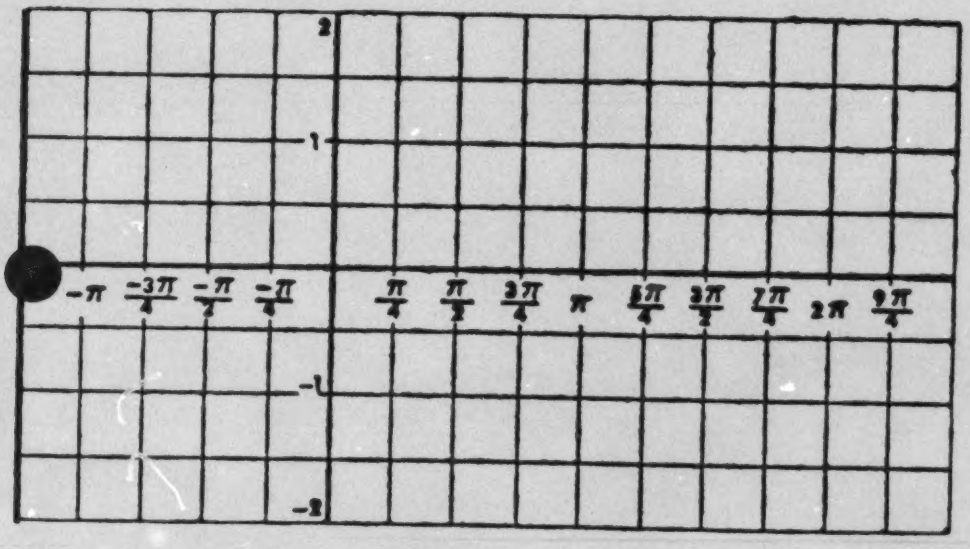
No solution: $a \leq b$

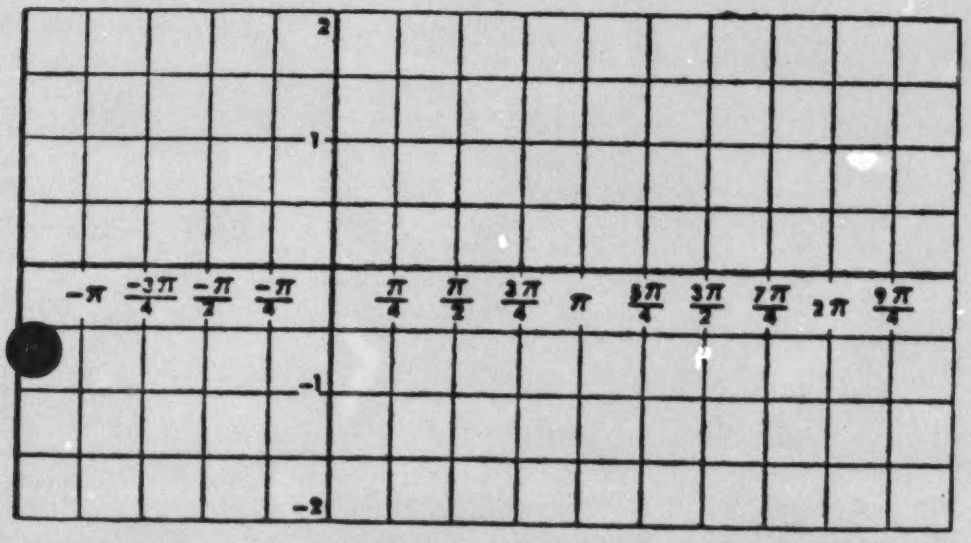


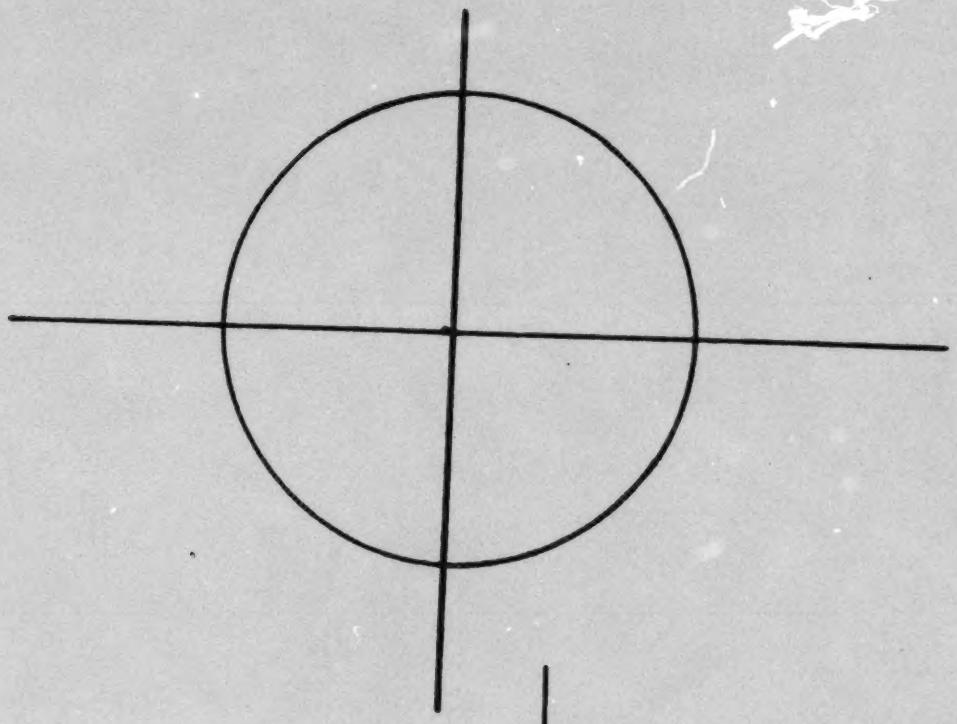
One solution $a > b$

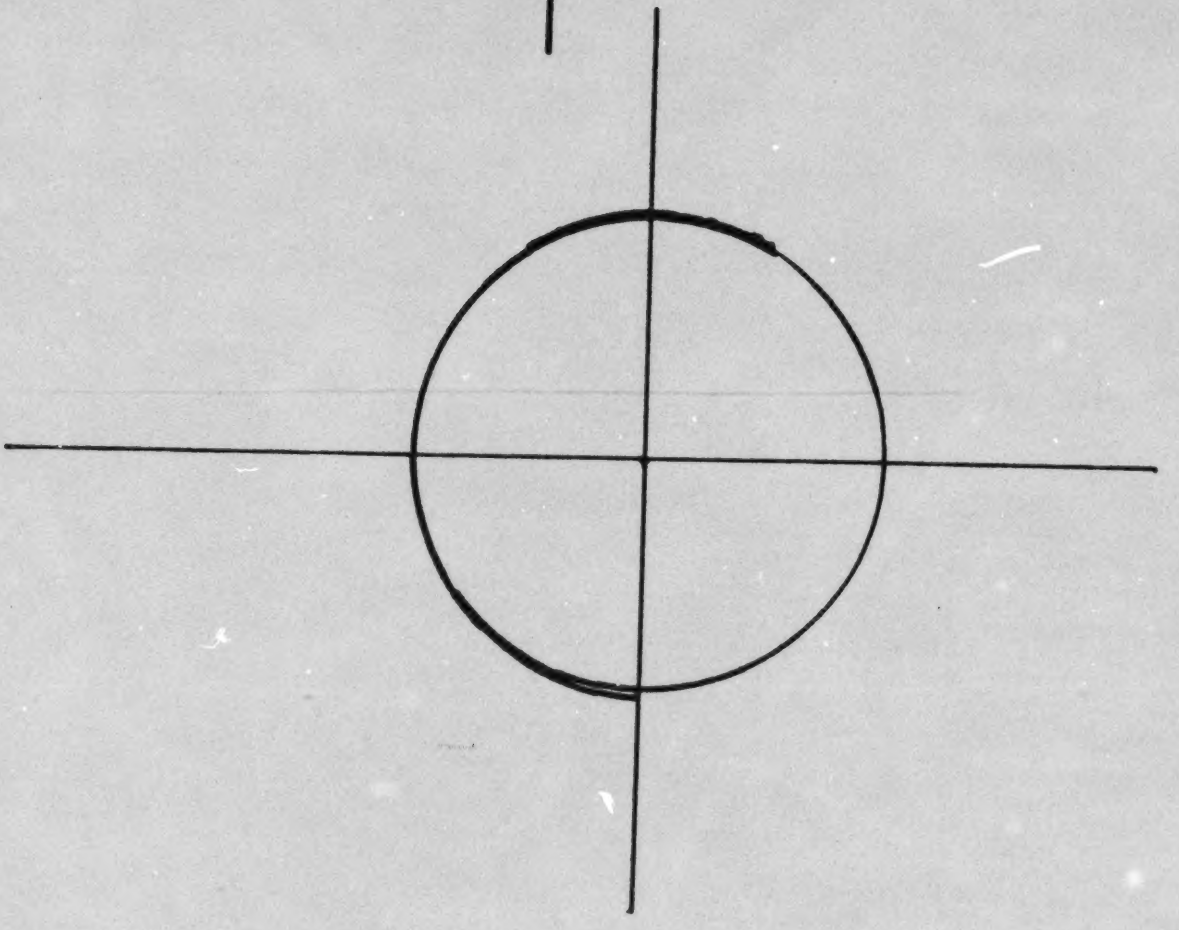
The Law of Sines may be used to solve the triangle for which the data set is ambiguous, but first you must determine whether a solution exists.


2
1
-1
-2
$-\pi$ $\frac{-3\pi}{4}$ $\frac{-\pi}{2}$ $\frac{-\pi}{4}$
$\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$ π $\frac{5\pi}{4}$ $\frac{3\pi}{2}$ $\frac{7\pi}{4}$ 2π $\frac{9\pi}{4}$

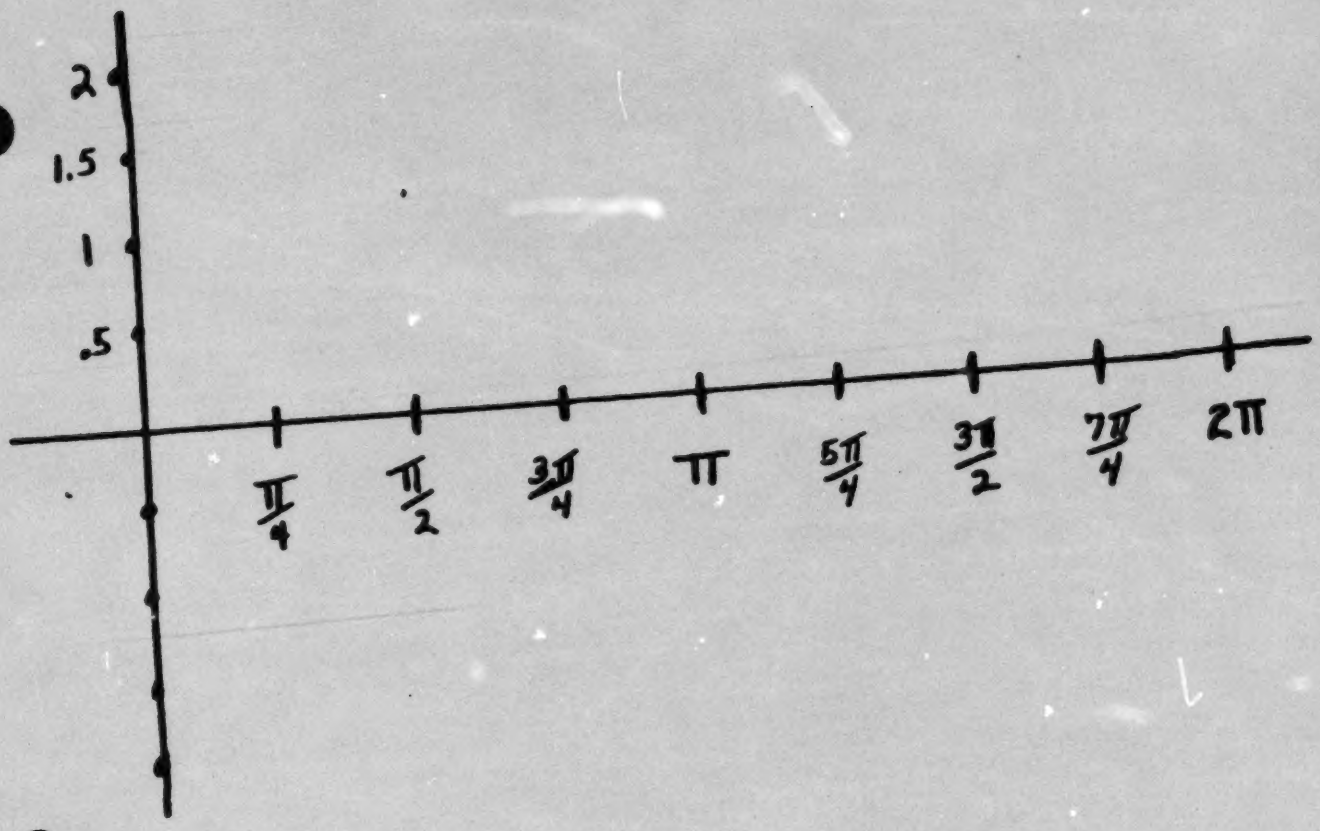

2
1
-1
-2
$-\pi$ $\frac{-3\pi}{4}$ $\frac{-\pi}{2}$ $\frac{-\pi}{4}$
$\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$ π $\frac{5\pi}{4}$ $\frac{3\pi}{2}$ $\frac{7\pi}{4}$ 2π $\frac{9\pi}{4}$


2
1
-1
-2
$-\pi$ $\frac{-3\pi}{4}$ $\frac{-\pi}{2}$ $\frac{-\pi}{4}$
$\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$ π $\frac{5\pi}{4}$ $\frac{3\pi}{2}$ $\frac{7\pi}{4}$ 2π $\frac{9\pi}{4}$



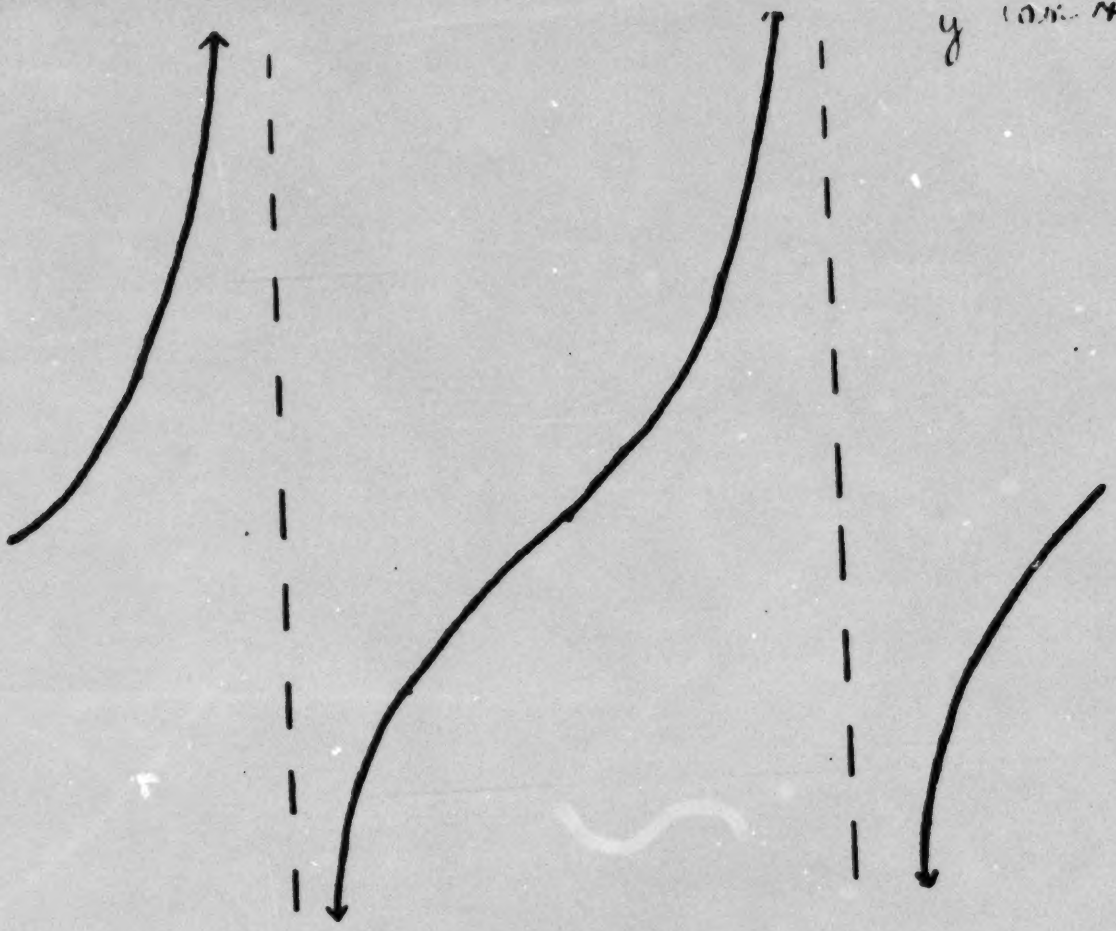


130


2
1.5
1
.5
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
$\frac{5\pi}{4}$
$\frac{3\pi}{2}$
$\frac{7\pi}{4}$
2π

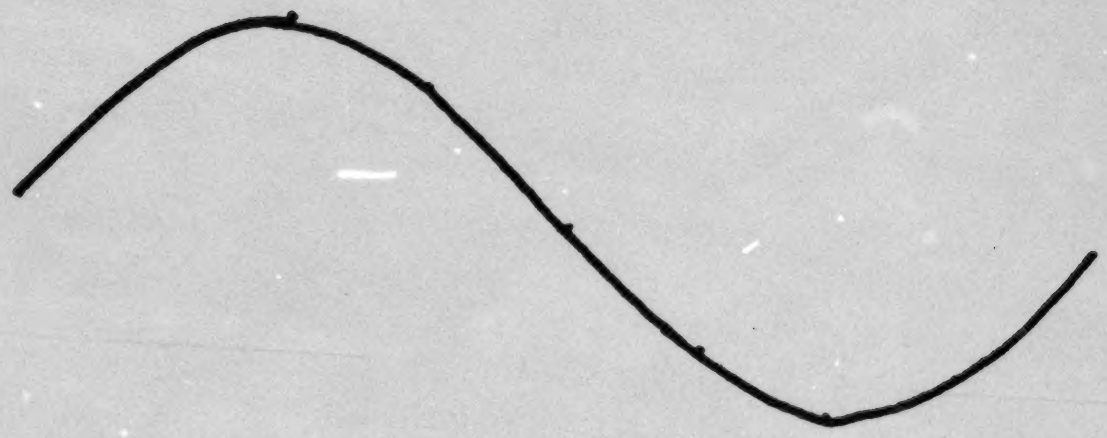
T7

131

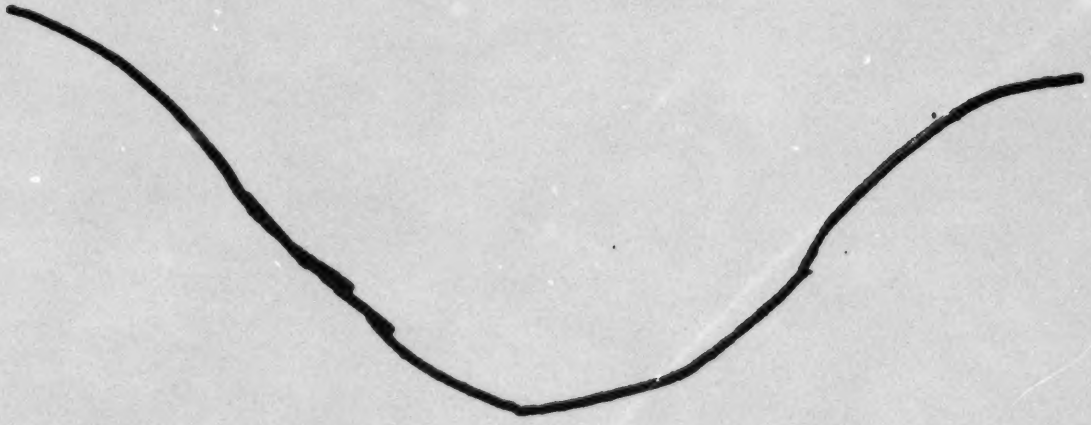

y

C.

$y = \sin x$ (use for $-\sin x$)

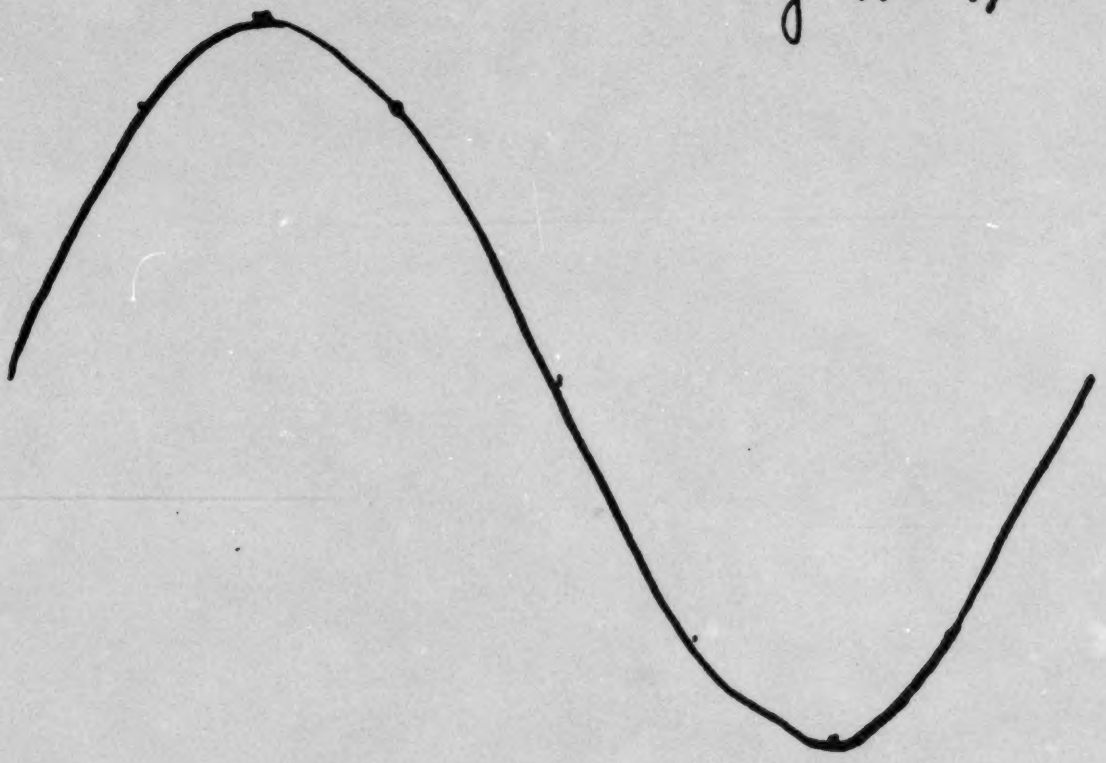


$y = \cos x$

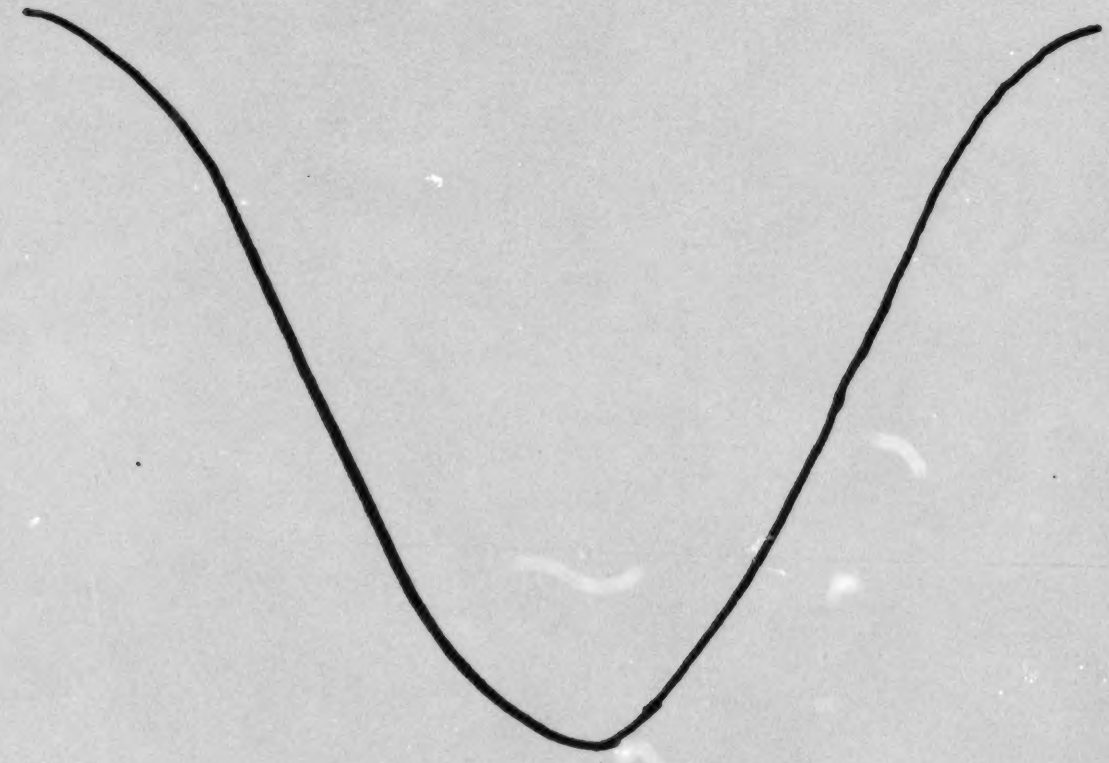


D.

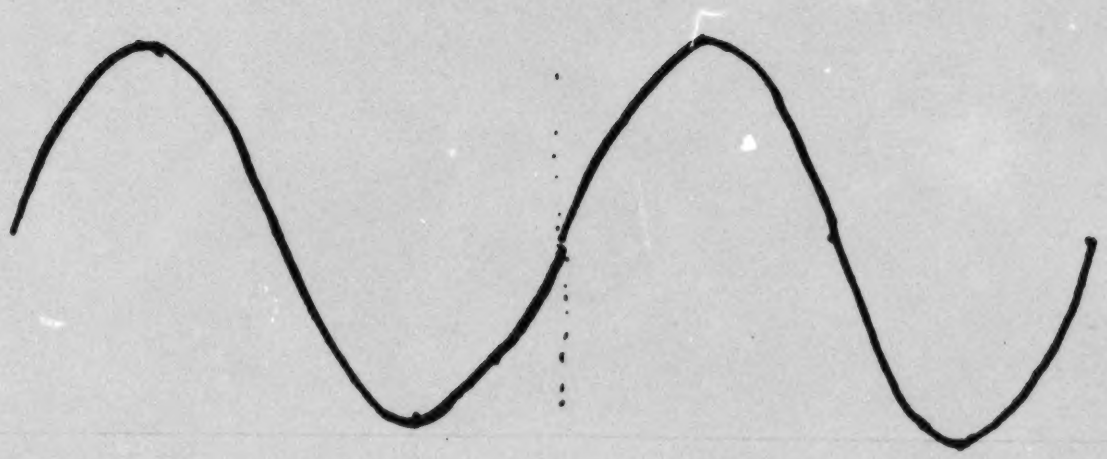
$y = 2 \sin x$



$y = 2\cos x$ D.



$y = \sin 2x$



T 11

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GEOMETRY OF THE CIRCLE

Angles of a Circle ---------- 123 - 129

a. Central and Inscribed Angles
b. Angles formed by two chords
c. Angles formed by a tangent and a chord
d. Angles formed outside of a circle
e. Angle Measurement

Length of Segments formed by: ---------- 130 - 134

a. Intersecting Chords
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Central and Inscribed Angles Name: _____

1. Find the measure of an inscribed angle which intercepts an arc whose measure is: a. 40 b. 80 c. 105 d. 120 e. a f. 4y

2. Find the measure of the arc intercepted by an inscribed angle whose measure is: a. 30 b. 75 c. $22\frac{1}{2}$ d. 45 e. b f. 6a

3. Inscribed angle ABC of circle O measures 75. Radii $\overline{AO}$ and $\overline{CO}$ are drawn. Find the measure of angle AOC.

4. AB is a diameter of a circle. $\overline{AC}$ is a chord, and arc $\overset{\frown}{AC}$ contained 100^0. Angle BAC contains _____ degrees.

5. An inscribed angle and a central angle intercept the same arc of a circle. The ratio of the measure of the inscribed angle to the measure of the central angle is _____.

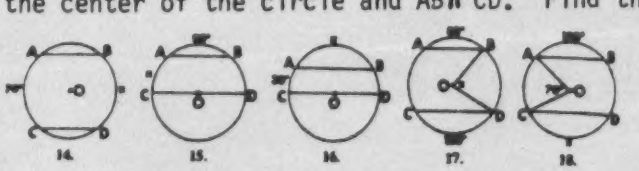
6. In circle O, central angle DOE measure 80. Find the measure of angle DFE, which is inscribed in minor arc $\overset{\frown}{DE}$.

7. Central angle AOB in circle O measures 120, and X is any point on minor arc $\overset{\frown}{AB}$. Find the measure of angle AXB.

In 8-12, O is the center of each circle. Find the value of x, or x and y.

13. In a circle, two parallel chords on opposite sides of the center have arcs which measure 100 and 120. Find the measure of one of the arcs included between the chords.

In 14-18, O is the center of the circle and $\overline{AB} \parallel \overline{CD}$. Find the value of x.



19. If quadrilateral ABCD is inscribed in a circle, find the measure of angle B if its opposite angle D measures: a. 65 b. 90 c. 110 d. a e. 180-2x

20. Quadrilateral ABCD is inscribed in a circle. If $m\overset{\frown}{AB} = 119$, $m\overset{\frown}{BC} = 73$, and $m\overset{\frown}{CD} = \overset{\frown}{50}$, what is the measure of angle ABC?

Angles Formed by Two Chords Name: _____

1. Find the measure of the angle formed by two chords intersecting within a circle if the opposite arcs they intercept measure: a. 40 and 60 b. 90 and 20 c. 110 and 44 d. 4x and 6x

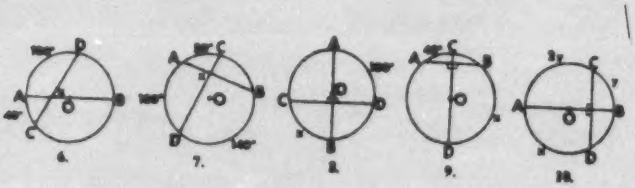
2. Find the sum of the measures of the opposite arcs intercepted by two chords which meet inside a circle if the chords form an angle whose measure is: a. 50 b. 70 c. 90 d. 37½ e. 67½ f. a + b

3. Chords $\overline{AB}$ and $\overline{CD}$ of a circle intersect at point E within the circle. If $m\overset{\frown}{AC}$ = 120 and $m\overset{\frown}{BD}$ = 80, find the measure of angle CEB.

4. Two chords intersecting within a circle form an angle whose measure is 60. If one of the intercepted arcs measures 80, what is the measure of the other intercepted arc?

5. In a circle chords $\overline{AB}$ and $\overline{CD}$ are perpendicular, and they intersect at E. If $m\overset{\frown}{AC}$ = 80, find the measure of arc $\overset{\frown}{BD}$.

In 6-10, O is the center of the circle. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside circle O. Find the value of x, or x and y, as indicated.



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Angles Formed by a Tangent and a Chord Name: _____

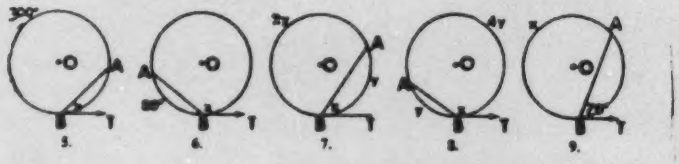
1. Find the measure of the angle formed by a tangent and a chord drawn from the point of contact if the angle intercepts an arc whose measure is: a. 40 b. 80 c. 65 d. 140 e. b f. 180 - 2x

2. Find the measure of the arc intercepted by an angle formed by a tangent and a chord drawn from the point of contact if the angle measures: a. 30 b. 50 c. $22\frac{1}{2}$ d. 70 e. m f. 3/2x

3. Triangle ABC is inscribed in a circle, side $\overline{AB}$ is a diameter of the circle, and arc $\overset{\frown}{AC}$ measures 100. The measure of the acute angle formed by the tangent at B and side $\overline{BC}$ of the triangle is _____.

4. Equilateral triangle ABC is inscribed in a circle. Find the measure of the acute angle formed by side $\overline{AB}$ and the tangent at B.

In 5 - 9, O is the center of the circle and $\overline{BT}$ is tangent to the circle. Find the value of x, or x and y, as indicated.



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Angles Formed Outside a Circle Name: _____

1. Find the measure of the angle formed by two tangents drawn to a circle from an external point if they intercept a minor arc whose measure is:
   a. 160 b. 140 c. 120
   d. 145 e. 135 f. 90

1. _____

2. If two tangents drawn from an external point to a circle intercept a major arc whose measure is 210, the angle between the tangents measures _____.

2. _____

3. If two tangents to a circle form an angle whose measure is 30, then the minor intercepted arc measures _____/

3. _____

4. Tangents $\overline{PA}$ and $\overline{PB}$ from an external point P to circle O form an angle whose measure is 70. If radii $\overline{OA}$ and $\overline{OB}$ are drawn, what is the measure of angle AOB?

4. _____

5. Find the measure of the angle formed by a tangent and a secant drawn to a circle from an external point if they intercept arcs whose measures are:
   a. 110 and 50 b. 120 and 65
   c. a and b d. 6a and 2a

5. _____

6. An angle formed by a tangent and a secant drawn to a circle from an external point measures 65. If the greater intercepted arc measures 170, find the measure of the smaller intercepted arc.

6. _____

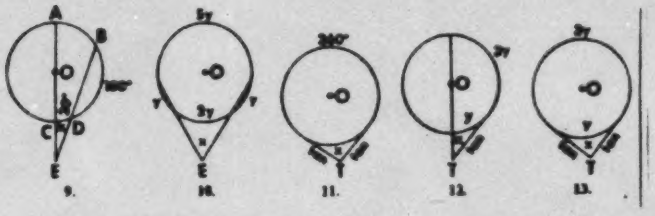
7. Find the measure of the angle formed by two secants drawn to a circle from an external point if they intercept arcs whose measures are:
   a. 100 and 40 b. 110 and 60
   c. m and n d. 7x and 3x

7. _____

8. From point P outside a circle, two secants $\overline{PAC}$ and $\overline{PBD}$. Angle P contains 32^0 and minor arc $\overset{\frown}{AB}$ containes 50^0. Find the number of degrees in minor arc $\overset{\frown}{CD}$.

8. _____

In 9-13, O is the center of the circle. Tangents to the circle are marked "tan". Find the value of x, or x and y, as indicated.



9. x = _____

10. x = _____
y = _____

11. x = _____

12. x = _____
y = _____

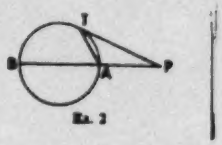
13. x = _____
y = _____

142

Angle Measurement Name: _____

1. Quadrilateral ABCD is inscribed in circle O; and the measures of arcs $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{CD}$, and $\overset{\frown}{DA}$ are in the ratio 2:5:3:8 respectively. (a) Find the measure of each arc. (b) Find $m\angle BAD$.

2. In the circle shown, $\overline{PT}$ is a tangent, $\overline{PAB}$ is a secant, and $\overline{TA}$ is a chord. $m\overset{\frown}{BT} = 112$, and $m\angle PTA = 34$.
   a. Find the measure of angle BAT.
   b. Find the measure of angle TPB.
   c. Show that $\overline{AB}$ is a diameter of the circle.

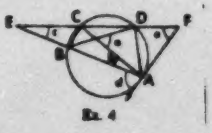


Ex. 2

3. Chord AC and diameter BD intersect at E. $m\overset{\frown}{AB} = 56$ and $m\angle BAC = 72$. Find the measures of $\overset{\frown}{AD}$, $\overset{\frown}{BC}$, angle BEC, and angle ABC.

Ex. 3

4. ABCD is a quadrilateral inscribed in a circle; $m\overset{\frown}{AB} = (4x - 25)$, $m\overset{\frown}{BC} = x$, $m\overset{\frown}{CD} = (2x + 20)$, $m\overset{\frown}{DA} = (3x-35)$, Chords $\overline{AC}$ and $\overline{BD}$ are drawn; also chords $\overline{AB}$ and $\overline{DC}$ are extended to intersect at E, and the tangent at A intersects $\overline{CD}$ extended at F. Find the number of degrees contained in angle a, angle b, angle c, angle d, and angle e.

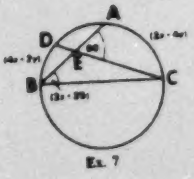


Ex. 4

5. Two secants $\overline{PDA}$ and $\overline{PCB}$ are drawn from external point P to a circle. Chords $\overline{AC}$ and $\overline{BD}$ intersect in F. If $m\overset{\frown}{AB} = 128$ and $m\overset{\frown}{CD} = 32$, find $m\angle P$, $m\angle ADB$, and $m\angle BFC$.

6. Congruent chords $\overline{AB}$ and $\overline{CD}$ of a circle are extended through B and D to intersect in P. $\overline{AD}$ and $\overline{BC}$ intersect in E, and $\overline{AC}$ is drawn. $m\angle P = 18$ and $m\overset{\frown}{AC} = 60$. Find $m\overset{\frown}{BD}$, $m\angle BCD$, $m\angle AEC$, and $m\angle ACB$.

7. Chords $\overline{AB}$ and $\overline{CD}$ intersect at point E, and chord $\overline{BC}$ is drawn. The measures of arcs $\overset{\frown}{AC}$ and $\overset{\frown}{BD}$ are represented by (2x + 4y) and (4x + 2y) respectively. The measure of angle ABC is represented by (2x + 25) and the measure of angle AEC is 60.
   a. In terms of x and y, write a set of equations that can be used to solve for x and y.
   b. Solve the set of equations written in answer to a to obtain the values of x and y.
   c. Find $m\overset{\frown}{AC}$ and $m\overset{\frown}{BD}$.



Ex. 7

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8. Secants $\overline{PQR}$ and $\overline{PST}$ are drawn to a circle from external point P so that $m\angle P = 20$ and $m\angle QTS = 10$. If $m\overset{\frown}{QS}$ is represented by $(3x + y)$ and $m\overset{\frown}{RT}$ by $(8x + 4y)$:
   a. Write a pair of equations that can be used to solve for x and y.
   b. Solve these equations to find values for x and y.
   c. Find $m\angle RQT$.

9. The sides of a triangle inscribed in a circle have arcs whose measures are represented by $(x + 15)$, $(6x + 10)$, and $8x - 40)$. Show that the triangle is an isosceles triangle.

10. The sides of a quadrilateral inscribed in a circle have arcs whose measures are represented by $(4x - 6)$, $(2x + 42)$, $(5x - 30)$, and $(3x + 18)$. Show that the quadrilateral is a square.

Length of Segments Formed by Intersecting Chords

Name:_____

1. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at E. AE = 4, CE = 3, and ED = 8. Find EB.

2. A diameter of a circle is perpendicular to a chord whose length is 14 inches. If the length of the shorter segment of the diameter is 4 inches, find the length of the longer segment of the diameter.

3. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside circle O at point E. If AE = 9 3/5, EB = 1 1/4, and DE = 3, find $\overline{EC}$.

4. Chords $\overline{AB}$ and $\overline{CE}$ intersect inside a circle at D. If AD = 1, CD = c, express DE in terms of A, b, and c.

5. In circle O, chord $\overline{AB}$ bisects chord $\overline{CD}$ at E. If AE = 16, and EB = 4, find the length of $\overline{CE}$.

6. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. If CD = 9, ED = 8, and AE is twice EB, find $\overline{EB}$.

7. A chord of a circle is perpendicular to a diamenter of the circle and divides the diameter into segments which are 2 inches and 18 inches in length. Find the length of the chord.

8. Point P is 3 inches from the center of a circle whose radius is 5 inches. (a) Find the legnth of the shortest chord that can be drawn through P. (b) Find the length of the longest chord which can be drawn through P.

9. In a circle, chord $\overline{CD}$, which is 16 in. in length, is drawn. The center of minor arc $\overset{\frown}{CE}$ is joined to the midpoint of chord $\overline{CD}$ by a line segment 4 in. in length. Find the length of the radius of the circle.

10. Chords $\overline{AB}$ and $\overline{CD}$ intersect inside a circle at point E. $\overline{AE}$ measures 4 in. $\overline{DE}$ measures 3 in, and $\overline{EB}$ measures 4 inches shorter than $\overline{CE}$. (A) If the length of $\overline{CE}$ is represented by x, write an equation that can be used to find x. (b) Find the length of the shorter chord.

11. In a circle, chords $\overline{AB}$ and $\overline{CD}$ intersect E. If CE = 6, ED = 4, AE = X + 3, and EB = x - 2, find $\overline{AE}$, $\overline{EB}$, and $\overline{AB}$.

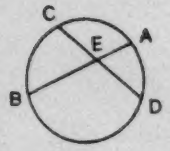
Intersecting Chords, etc.

In 12-24, chords $\overline{AB}$ and $\overline{CE}$ intersect at E.

12. If CE = 12, ED = 2, and AE = 3, find EB.

13. If CE = 18, ED = 2, and BE = 6, find AE.

14. If AE = 8, EB = 9, and CE = 12, find ED.



15. If CE = 18, ED = 6, and AE = 4, find EB.

16. If AB = 9, EA = 4, and CE = 2, find ED.

17. If AB = 20, BE = 8, and CE = 16, find ED.

18. If AE = 9, EB = 16, and CE = ED, find CE.

19. If EB = 8, ED = 10, and AE is 1 more than CE, find CE.

20. If AE = 3, CE = 5, and ED is 4 less than EB, find EB.

21. If CE = 4, ED = 12, and EB is 2 more than AE, find AE.

22. Chords $\overline{CE}$ and $\overline{JK}$ intersect at point E within a circle. If CE = 10, ED = 6, and JE = 4, find EK.

23. Chords $\overline{AB}$ and $\overline{CD}$ intersect at point E within a circle; AE = 9 and EB = 1.
    a. If CE = 6 and CE = x, what expresseniton represents ED in terms of x.
    b. Find CE.
    c. Find ED.

24. If chords $\overline{AB}$ and $\overline{RS}$ intersect at point E within a circle so that AE = 3, EB = 16, and RE:ES = 3:4, find: a. RE b. ES c. RS

Lengths of Segments
formed by Intersecting Secants

Name: _____

In 1-7, secants $\overleftrightarrow{ABC}$ and $\overleftrightarrow{ADE}$ intersect at point A outside the circle.

1. If AC = 15, AB = 6, and AE = 10, find AD.

2. If AC = 20, AB = 5, and AD = 4, find AE.

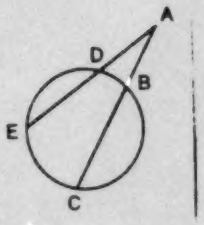
3. If AB = 9, BC = 11, and AD = 10, find AE.

4. If AB = 3, BC = 7, and AD = 5, find DE.

5. If AB = 2, AD = 3, and DE = 5, find BC.

6. If AD = 6, AE = 10, and AC = 12, find:
   a. AB b. BC

7. If AC = 18, AB = 4, and ED = 1, find AD.



8. Two secants are drawn to a circle from an outside point. The length of one secant is 20 and the length of its external segment is 5. If the length of the other secant is 25, find the length of its external segmant.

9. Two secants are drawn to a circle from an outside point. The external segment of the first secant is 4 and its internal segment is 8. Find the length of the second secant if its external segment is 6.

Lengths of Segments Formed By a Secant and a Tangent

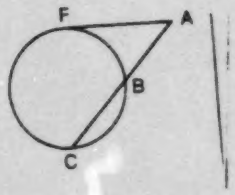
Name:_____

In 1-8, $\overleftrightarrow{AF}$ is tangent to the circle at F, and secant ABC intersects the circle of B and C.

1. If AF = 8, and AB = 4, find AC.

2. If AC = 12 and AB = 3, find AF.

3. If AF = 6 and AC = 9, find AB.



4. If AB = 2 and BC = 6, find AF.

5. If AB = 4 and BC = 21, find AF.

6. If AC:ab = 4:1 and AF = 12, find AB.

7. If AB:BC = 1:3 and AF = 4, find AB.

8. If AF = 10 and BC = 15, find AB.

9. Secant $\overleftrightarrow{ABC}$ and tangent $\overleftrightarrow{AD}$ intersect at A outside the circle, while B, C, and D are points on the circle. If AB = 16 and BC = 9, find the length of the tangent segment AD.

10. In a circle, diameter $\overline{AB}$ is extended through B to an external point P, and tangent segment PC is drawn to point C on the circle. If BP = 4 and PC = 6, find the length of $\overline{AB}$.

11. If the length of a secant to a circle from an external point is 9 and the length of its external segment is 4, the length of the tangent from that point is _____.

12. A tangent and a secant are drawn to a circle from the same point. If the length of the external segment of the secant is 3 and the legnth of the internal segment is 7, find the length of the tangent.

Lengths of Segments, etc.

13. The diameter of a circle is 15 inches. If this diameter is extended 5 inches beyond the circle to point A, find the number of inches in the length of a tangent to the circle from point A.

14. The length of a tangent drawn from a point 3 inches from a circle the length of whose radius is 12 inches is _____ inches.

15. Point P is 15 in. from the center of a circle the length of whose radius is 9 in. Find the length of the tangent drawn from point P to the circle.

16. Diameter $\overline{AB}$ of circle O is 21 inches in length. $\overline{AB}$ is extended through B to a point C and tangent $\overline{CD}$ is drawn, meeting the circle at D. $\overline{CD}$ is 6 inches longer than $\overline{BC}$.

    a. Let x represent the length of $\overline{BC}$, and express CD and CA in terms of x.
    b. Express as an equation involving the variable x the relationship that exists among BC, CA and CD.
    c. Find the length of $\overline{BC}$ by solving the equation obtained in part b.

Intercepted Arcs Name: _____

1. In a circle whose circumference is 36 inches, find the length of an arch which contains:
   a. 30^0 b. 60^0 c. 90^0 d. 240^0 e. 45^0

2. Find the circumference of a circle if the length of an arc of the circle which contains 90^0 is 5 inches.

3. In a circle whose radius is 12, find the length of an arch whose central angle contains:
   a. 60^0 b. 90^0 c. 150^0 d. 240^0 e. 50^0

4. In a circle whose radius is 4 inches, find to the nearest inch, the length of the minor arc intercepted by a chord 6 inches in length.

5. In a circle whose radius is 20 inches, find to the nearest inch the length of the minor arc intercepted by a chord 24 inches in length.

6. In a circle whose circumference is 24 inches, find the measure of the central angle of an arc whose length is 8 inches.

7. In a circle whose radius is 16, find the measure of the central angle of an arc whose length is: a. 4π b. 8π c. 12π d. 16π e. 20π

8. Find the radius of a circle in which an arch which contains 60^0 has the length of:
   a. 3π b. 5π c. 6π d. 8π e. 12π

9. Find the length of an arc intercepted by a side of a regular hexagon inscribed in a circle whose radius is 18.

Area of Sectors and Segments Name: _____

1. Find what fractional part of the area of a circle the area of a sector is if its central angle contains:
   a. 90^0 b. 30^0 c. 135^0 d. 240^0 e. 40^0

2. If the central angle of a sector of a circle contains 40^0, the area of the circle is _____ times the area of the sector.

3. If the area of a sector of a circle is to the area of the circle as 1:8, then the number of degrees contained in the angle of the sector is _____.

4. The area of a circle is 80 square inches. Find the area of a sector whose central angle contains 45^0.

5. In a circle whose radius is 12, find the area of a sector whose central angle contains:
   a. 90^0 b. 45^0 c. 60^0 d. 120^0 e. 80^0

6. The radius of a circle is 9 and the angle of a sector of this circle measures 40. The area of this sector, interms of π, is _____.

7. Find the radius of a circle in which a sector whose central angle measures 90 has an area of: a. 4π b. 25π c. 36π d. 9π e. π

8. The angle of a sector of a circle measures 120 and the area of the sector is 27π. Find the radius of the circle.

9. In a circle whose radius is 12, find the measure of the central angle of a sector whose area is: a. 36π b. 12π c. 72π d. 16π e. 60π

10. $\overline{BA}$ and $\overline{BC}$ are tangents to circle O at A and C respectively, and form an angle of 50^0. Line segments $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ are drawn. The length of $\overline{OB}$ is 14.2 inches.
    a. Find to the nearest inch the length of the radius of the circle.
    b. Using the value of the radius obtained in answer a and using $\pi = \frac{22}{7}$ find to the nearest square inch the area of minor sector AOC.

In 11 and 12, the answers may be left in radical form and in terms of π.

11. In a circle whose radius is 6, find the area of a minor segment whose arc has a central angle which contains:
    a. 90^0 b. 60^0 c. 30^0 d. 120^0 e. 140^0

12. In a circle, a chord is drawn whose length is equal to the length of a radius. Find the area of the minor segment formed by the chord and its arc if the length of a radius of the circle is: a. 6 b. 18 c. 4 d. 10 e. 5

In this exercise, use $\pi = 3.14$, $\sqrt{3} = 1.73$, and express each result to the nearest inch.

13. In circle O, chord $\overline{AB}$ and a radius are each 6 inches in length. If radii $\overline{OA}$ and $\overline{OB}$ are drawn, find: (a) the length of minor arc $\overset{\frown}{AB}$. (b) the area of triangle AOB. (c) the area of sector AOB. (d) the area of the minor segment of the circle.

14. A man has a flower garden in the shape of a minor segment of a circle. The arc of the segment contains 90^0 and the radius of the circle is 28 feet long. (Use $\pi = \frac{22}{7}$). (a) Find to the nearest foot the number of feet of fencing required to enclose the garden. (b) Find the area of the garden.

15. In circle O, central angle AOB contains 90^0, the length of AB = 6 and chord $\overline{AB}$ is drawn. (a) Find the length of radius $\overline{OA}$. (b) Find the area of triangle AOB. (c) Find the area of sector AOB. (Answer may be left in terms of π). (d) Find the area of minor segment AB. (Answer may be left in terms of π).

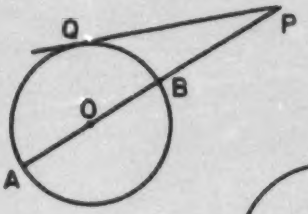
16. Find the sum of the areas of the four segments formed by inscribing a square in a circle whose radius is 8.

Regents Review Name: _____

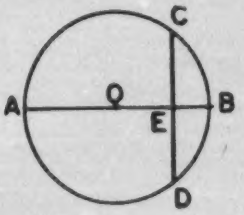
1. Chords $\overline{AB}$ and $\overline{CE}$ of circle O intersect at E. If AE = EB = 4 and CE = 8, find ED.

2. In a circle, a central angle of 3 radians intercepts an arc of 18 centimeters. What is the radius, in centimeters, of the circle?

3. In the accompanying diagram, circle O has radius 6, diameter $\overline{AB}$ is extended through B to P, tangent $\overline{PQ}$ is drawn, and PQ = 8. Find the length of $\overline{BP}$.



4. In the accompanying diagram of circle O, diameter $\overline{AB} \perp \overline{CD}$, and CD = 14. Find CE.



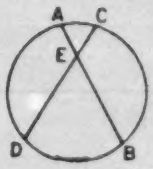
5. Given a circle with radius of 6 centimeters. Find the length in centimeters of the arc intercepted by a central angle of 1 1/3 radians.

6. Two tangents $\overline{PA}$ and $\overline{PB}$ are drawn to circle O from an external point P. If the measure of major arc $\overset{\frown}{AB}$ is 250^0, find the measure of $\angle P$.

7. Given: circle O with $\overline{QR}$ tangent to the circle at R. If m $\angle$OQR = 20, find m$\angle$ ROQ.

8. If circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If AE = 15, EB = 5, and CE = 25, find ED.

9. In the accompanying figure, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If m$\overset{\frown}{AC}$ = 30 and m$\overset{\frown}{BD}$ = 90, find m$\angle$AEC.



10. In a circle of radius 6, find the length of the arc intercepted b a central angle of 2 radians.

11. Chords $\overline{AB}$ and $\overline{CD}$ of circle O intersect at E. If AE = 4, EB = 5, and CE = 2, find ED.

12. In the accompanhing figure, $\overline{AD}$ is tangent to the circle at D and $\overline{ABC}$ is a secant. If m$\overset{\frown}{DC}$ = 150 and m$\overset{\frown}{BD}$ = 90, find m $\angle$ A.

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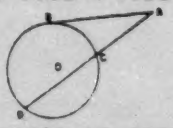
13. In a circle, a central angle of 2 radians intercepts an arc of 10 centimeters. What is the length, in centimeters, of the radius of the circle?

14. In a circle with radius 4, find the number of radians in a central angle which intercepts an arc 10 units long.

15. In the accompanying diagram, tangent AB and secant $\overline{ACD}$ are drawn to circle O. If AC = 4 and CD = 5, find AB.



16. In a circle, a central angle of 2 radians intercepts an arc of length 12 centimeters. Find the length of the radius in centimeters.

17. In circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If AE = 6, BE = 12, and CE = 8, find DE.

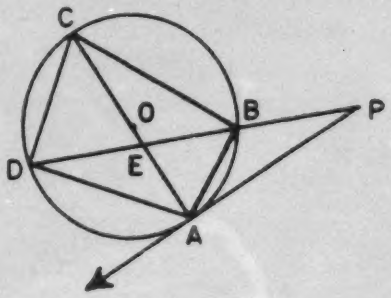
18. Two tangents drawn to a circle from an external point are perpendicular. Find the measure of the <u>smaller</u> of the intercepted arcs.

19. In a circle with radius 4.5 centimeters, find, in centimeters, the length of the arc intercepted by a central angle of 3 radians.

20. Chords $\overline{AB}$ and $\overline{CD}$ intersect in a circle at point E. If AE = 6, EB = 3, and ED = 2, find CE.

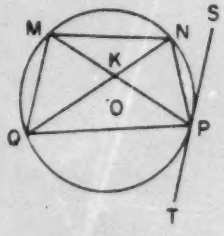
21. In the accompanying figure, PA is tangent to circle O at A, secant $\overline{PBD}$ intersects diameter $\overline{AC}$ at E, $m\angle P = 25$, and $m\overset{\frown}{AD} = 106$.

Find:
a. $m\overset{\frown}{AB}$
b. $m\overset{\frown}{BC}$
c. $m\angle PAB$
d. $m\angle BEA$
e. $m\angle CAD$



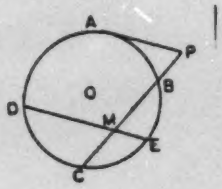
22. In the accompanying figure, quadrilateral MNPQ is inscribed in circle O with $\overline{ST}$ tangent to the circle at P. Diagonals $\overline{PM}$ and $\overline{QN}$ intersect at K and $m\overset{\frown}{MN} : m\overset{\frown}{NP} : m\overset{\frown}{PQ} : m\overset{\frown}{QM} = 5:3:8:4$.

Find:
a. $m\overset{\frown}{MN}$
b. $m\angle MQP$
c. $m\angle SPQ$
d. $m\angle PKQ$

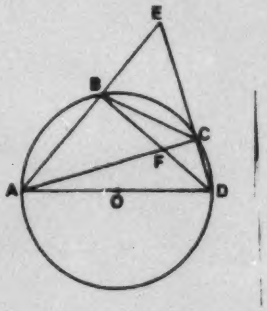


Regents Review - Part II

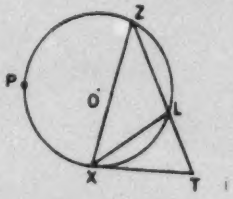
1. Given: circle O with tangent $\overline{PA}$ and secant $\overline{PBC}$, chord $\overline{DE}$ bisects chord $\overline{BC}$ at M and PA = 3 and PB = 1.
   a. Find BC.
   b. If DE = 10 and DM > EM, find EM.



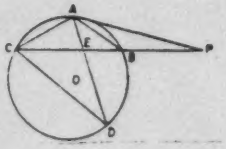
2. In the accompanying figure, quadrilateral ABCD is inscribed in circle O. Diagonals $\overline{AC}$ and $\overline{BD}$ meet at F, and $\overline{AD}$ is a diameter. Chords $\overline{AB}$ and $\overline{DC}$ are extended to meet at E. B is the midpoint of $\overline{AC}$ and $m\overset{\frown}{AB}:m\overset{\frown}{CD} = 4:1$.
   Find:
   a. $m\overset{\frown}{CD}$
   b. $m\angle BDA$
   c. $m\angle BFC$
   d. $m\angle E$
   e. $m\angle EBD$



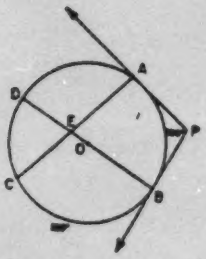
3. Given: circle O, tangent $\overline{TX}$, secant $\overline{TLZ}$, chords $\overline{ZX}$ and $\overline{XL}$, $m\overset{\frown}{XL}:m\overset{\frown}{LZ}:m\overset{\frown}{XPZ} = 2:2:5$.
   Find:
   a. $m\overset{\frown}{XL}$
   b. $m\angle Z$
   c. $m\angle T$
   d. $m\angle ZXT$
   e. $m\angle XLT$



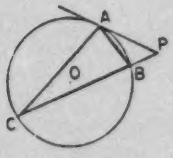
4. In the accompanying diagram, $\overline{PA}$ is tangent to circle O at A and secant $\overline{PC}$ intersects circle O at B and C. Chord $\overline{AD}$ intersects $\overline{BC}$ at E, $m\overset{\frown}{AC} = 82$, $m\angle P = 14$, and $m\overset{\frown}{CD}:m\overset{\frown}{BD} = 5:3$.
   Find:
   a. $m\overset{\frown}{AB}$
   b. $m\overset{\frown}{BD}$
   c. $m\angle PAB$
   d. $m\angle BCD$
   e. $m\angle CED$



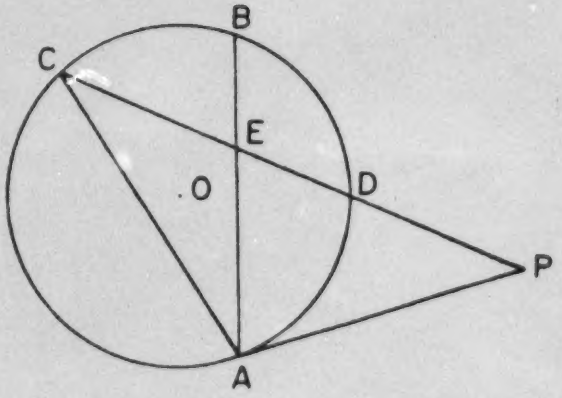
5. In the accompanying diagram, $\overline{PA}$ and $\overline{PB}$ are tangent to circle O at A and B, resepctively, Diameter $\overline{BD}$ and chord $\overline{AC}$ intersect at E, with $m\overset{\frown}{CB} = 120$ and $m\angle APB = 100$.
   a. Find $m\overset{\frown}{AB}$
   b. Find $m\overset{\frown}{AD}$
   c. Find $m\angle DEC$
   d. Find $m\angle PAC$



6. In the accompanying diagram, $\overline{PA}$ is tangent to circle O at A, $\overline{PBC}$ is a secant, $m\angle P = 42$, and $m\overset{\frown}{AC} : m\overset{\frown}{AB} = 5:2$.
   a. Find $m\overset{\frown}{AC}$
   b. Find $m\overset{\frown}{BC}$
   c. Find $m\angle ACB$
   d. Find $m\angle ABP$



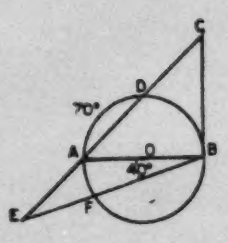
7. In the diagram below, $\overline{PA}$ is tangent to circle O at A and secant $\overline{PC}$ intersects circle O at C and D. Chord $\overline{CA}$ is drawn, chords $\overline{BA}$ and $\overline{CD}$ intersect at E, $m\overset{\frown}{BC} = 50$, $m\overset{\frown}{BEC} = 60$, $m\overset{\frown}{BD} = x$, and $m\overset{\frown}{CA} = 2x + 30$.
   a. Find $m\overset{\frown}{DA}$ [3]
   b. $m\overset{\frown}{BD}$ [3]
   c. $m\angle CPA$ (2)
   d. $m\angle BAC$ [2]



Regents Review - Part II

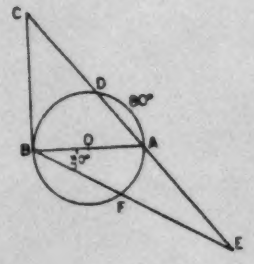
Name: _____

1. In the accompanying diagram, $\overset{\frown}{AOB}$ is a diameter of circle O, $\overline{CB}$ is tangent to the circle at B, $\overline{BAC}$ and $\overline{EFB}$ are secants, $m\overset{\frown}{AD} = 70$, and $m\angle ABF = 40$.



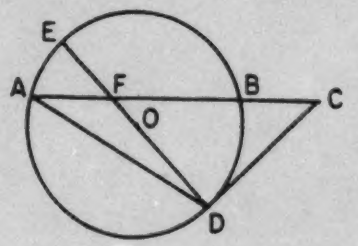
Find:
   a. $m\angle CAB$ [2]
   b. $m\overset{\frown}{AF}$ [2]
   c. $m\angle E$ [2]
   d. $m\angle C$ [2]
   e. $m\angle ABC$ [2]

2. In the accompanying diagram, $\overline{AOB}$ is a diameter of circle O, $\overline{CB}$ is tangent to the circle at B, $\overline{EAC}$ and $\overline{EFB}$ are secants. $m\overset{\frown}{AD} = 80$, and $m\angle ABF = 30$.



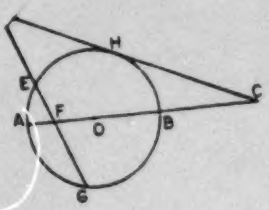
   a. Find $m\angle CAB$ [2]
   b. Find $m\angle E$ [2]
   c. Find $m\angle C$ [2]
   d. Find $m\angle ABC$ [2]

3. In the accomapnying diagram, $\overline{CD}$ is tangent to circle O, secant $\overline{ABC}$, chord $\overline{AB} \cong \overline{AD}$, $m\overset{\frown}{AB} = 150$, and chord $\overline{EOD}$ is a diameter.



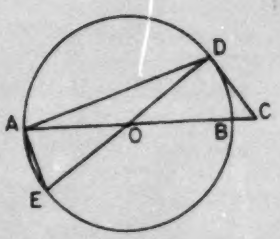
   a. Find $m\overset{\frown}{AD}$ [2]
   b. Find $m\angle BAD$ [2]
   c. Find $m\angle ACD$ [2]
   d. Find $m\angle EFB$ [2]
   e. Find $m\angle EDA$ [2]

4. In the accompanying diagram $\overline{DC}$ is tangent to circle O at H, $\overline{AFOBC}$, secant $\overline{DEC}$ intersects secant $\overline{CBA}$ at F, $m\overset{\frown}{AEH} = 110$, $m\overset{\frown}{AC} = 80$, and the ratio between $m\overset{\frown}{AE}$ and $m\overset{\frown}{EH}$ is 2:3.



   a. Find $m\overset{\frown}{CB}$ [1]
   b. Find $m\overset{\frown}{EA}$ [3]
   c. Find $m\angle EFA$ [3]
   d. $m\angle DCA$ [3]

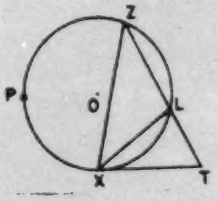
5. In circle O, diameter $\overline{AB}$ is extended to point C, $\overline{CD}$ is tangent to the circle at D, $\overline{DE}$ is a diameter and $m\overset{\frown}{BD}$: $m\overset{\frown}{AD} = 14$.



Find:

a. $m\overset{\frown}{BD}$ [2]
b. $m\angle E$ [2]
c. $m\angle C$ [2]
d. $m\overset{\frown}{AE}$ [2]
e. $m\angle ADE$ [2]

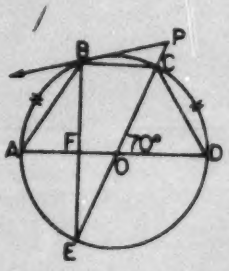
6. Given circle O, tangent $\overline{TX}$, secant $\overline{TLZ}$, chords $\overline{ZX}$ and $\overline{XL}$, $m\overset{\frown}{XL} : m\overset{\frown}{LZ} : m\overset{\frown}{XPZ} = 2:2:5$



Find:

a. $m\overset{\frown}{XL}$ [2]
b. $m\angle Z$ [2]
c. $m\angle T$ [2]
d. $m\angle ZXT$ [2]
e. $m\angle XLT$ [2]

7. In the accompanying diagram, ABCD is inscribed in circle O, $m\angle COD = 70$, $\overline{AOD}$ is a diameter. $\overline{PB}$ is tangent to circle O at B, $\overline{PCOE}$ is a secant and $m\overset{\frown}{BA} = m\overset{\frown}{CD}$.



Find:

a. $m\overset{\frown}{BC}$ [2]
b. $m\angle PBC$ [2]
c. $m\angle A$ [2]
d. $m\angle BPE$ [2]
e. $m\angle AFE$ [2]

COURSE III - Geometry Unit - Circles

A. Basic Definitions
1. Circle
2. Center
3. Radius
4. Equal Circles
5. Concentric Circles
6. Chord
7. Diameter
8. Secant
9. Tangent Line
10. Circumscribed, inscribed
11. Central Angle
12. Minor and Major arc, semi circle

B. Circumference, Area and Pi

1. Area of a sector
2. Length of an arc
3. Apothem: $A = \frac{1}{2} ap$

C. Tangent Theorems

1. The $\perp$ segment from a point to a line is the shortest segment from the point to the line.
2. A line tangent to a circle is $\perp$ to the radius drawn to the peini ci tangency and concisely.
3. Tangents to a circle from an outside point are equal.
4. Externally and internally tangent circles.
5. Line of centers
6. Common external and internal tangents.

D. Chord Theorems

1. In the same circle, or in equal circles, equal chords have equal arcs and conversely.
2. A line passing through the center of a circle and $\perp$ to a chord bisects the chord and its arc.
3. In the same circle or in equal circles, equal chords are equally distant from the center and conversely.

E. Inscribed Angle Theorems

1. Definition
2. An inscribed angle is measured by $\frac{1}{2}$ its intercepted arc.
3. If 2 chords intersect within a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other.

F. Angles Formed by Chords, Tangents, Secants

1. An angle formed by 2 chords intersecting within a circle is measured by ½ the sum of the intercepted arcs.
2. An angle formed by a chord and a tangent is measured by ½ the intercepted arc.
3. An angle formed by 2 secants intersecting outside a circle is measured by ½ the difference of the intercepted arcs.
4. The measure of an angle formed by a secant and a tangent intersecting outside a circle is measured by ½ the difference in measures of the intercepted arcs.
5. The angle formed by 2 tangents to a circle is measured by ½ the difference in the measures of the intercepted arcs.
6. If 2 secants are drawn to circle from an outside point, the product of the lengths of one secant and its external segment equals the product of the lengths of the other secant and its external segment.
7. If a tangent and a secant are drawn to a circle from an outside point, the square of the length of the tangent segment is equal to the product of the lengths of the secant and its external segment.

TRANSFORMATIONAL GEOMETRY
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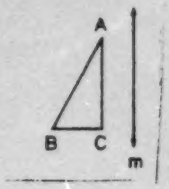
Reflections - Euclidean Plan

Sketch, as carefully as possible, the image of the given figure under a reflection in line m. Label image points as in A A'.

1.



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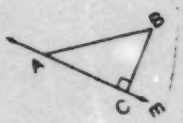

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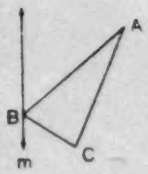

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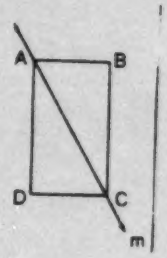

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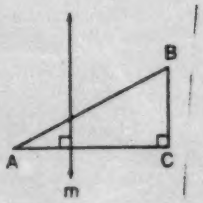

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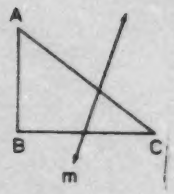
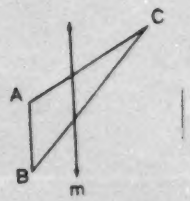


8.



Reflections - Coordinate Plane Name: _____

In 1 and 2, find the image point under a feflection in the x-axis.

1. (5,7) _____ 2. (-1, -4) _____

In 3 and 4, find the image of the point under a reflection in the y-axis.

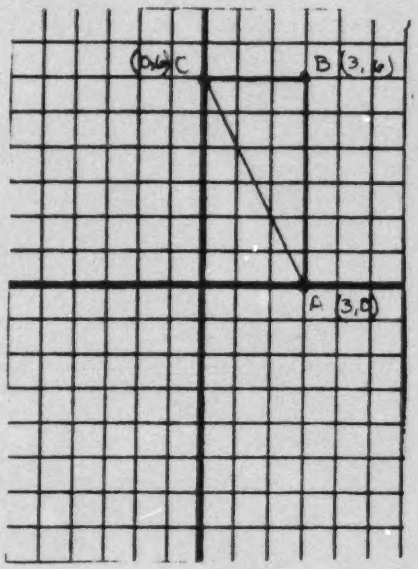
3. (5, 7) _____ 4. (0, 6) _____

In 5-7, find the image of the point under a reflection in the line, y = x.

5. (5, 7) _____ 6. (-3, 8) _____ 7. (0, 2) _____

In 8-10, the vertices of △ABC are A (3,0), B(3, 6) and C(0,6).

8. △ABC is reflected in the x-axis.
   a. Find the coordinates of the images of the vertices, namely, A', B', and C'.
   b. On one set of axes, draw △ABC and △A'B'C'.
   c. Find the coordinates of all points on the sides of △ABC that remain fixed under the given reflection.



9. △ABC is reflected in the y-axis. Answer again parts a, b, and c of exercise 8 for this line reflection.

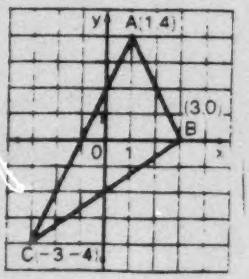
10. △ABC is reflected in the line y = x. Answer again parts a, b, and c of exercise 8 for this line reflection.

In 11-13, the vertices of △ABC are A (1, 4), B(3, 0), and C(-3, -4).

11. △ABC is reflected in the x-axis. Answer again parts a, b, and c of exercise 8 for this line reflection.

12. △ABC is reflected in the y-axis. Answer again parts a, b, and c of exercise i for this line reflection.

13. △ABC is reflected in the line y = x. Answer again parts a, b, and c of exercise 8 for this line reflection.



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Reflections - Coordinate Plane

In 14-20, the image of △ABC under a line reflection is △A'B'C'.

a. Using the given coordinates, draw △ABC and △A'B'C' on one set of axes.

b. Find the equation of the line of reflections.

14. △ABC: A(2,4), B(2,1), and C(-1,1).
△A'B'C': A'(4,4), B'(4, 1), and C'(7,1).

15. △ABC: A(1,3), B(2,5), and C(5,3).
△A'B'C': A'(1,1), B'(2, -1), and C'(5,1).

16. △ABC: A(1,4), B(2,1), and C(4,2).
△A'B'C': A'(-5,4), B'(-6, 1), and C'(-8,2).

17. △ABC: A(4,2), B(6,2), and C(2,-1).
△A'B'C': A'(2,4), B'(2, 6), and C'(-1,2).

18. △ABC: A(-1,0), B(0,2), and C(4,1).
△A'B'C': A'(-1,8), B'(0, 6), and C'(4,7).

19. △ABC: A(0,1), B(3,1), and C(2,5).
△A'B'C': A'(-1,2), B'(-1, 5), and C'(2,5).

20. △ABC: A(2,-1), B(4,2), and C(-1,2).
△A'B'C': A'(1,-2), B'(-2,-4), and C'(-2,1).

21. The vertices of △DEF are D(3,4), E(4,1) and F(6,3). If △DEF is reflected in the line whose equation is x = 2, find the coordinates of the images D', E', and F'.

22. The vertices of △RST are R(0,0), S(1,2), and T(4,1). if △RST is reflected in the oine whose equation is y = 3, find the coordinates of the images R', S', and T'.

Translations Name: _____

In 1-4, use the rule $(x, y) \rightarrow (x - 3, y + 12)$ to find the image of the given point.

1. (4, 4) 2. (2, 0) 3. (-3, -15) 4. (-5, -5)

In 5-8, find the image of the point (2, 7) under the given translation.

5. $T_{1,2}$ 6. $T_{3, -6}$ 7. $T_{-4, 0}$ 8. $T_{-2, -8}$

In 9-14, find the rule for the translation so that the image of A is A'.

9. $A(3, 8) \rightarrow A'(4, 6)$
10. $A(1,0) \rightarrow A'(0,1)$
11. $A(2,5) \rightarrow A'(-1, 1)$
12. $A(-1,2) \rightarrow A'(-2, -3)$
13. $A(0, -3) \rightarrow A'(-7, -3)$
14. $A(4, -7) \rightarrow A'(4, -2)$

15. A translation maps B(0,2) onto B'(5,0). Find the coordinates of C', the image of C(-3,1), under the same translation

16. A translation maps the origin to the point (-1, 7). What is the image of (3, -7) under the same translation?

17. The endpoints of $\overline{AB}$ are A(3,4) and B(5,-2). Find the rule for the translation that maps A onto the midpoint of $\overline{AB}$.

18. Under a given translation, the image of (3,3) is (6,1). Using the same translation, find the preimage of: a. (8,3) b. (3,-2) c. (2,-5).

19. a. On graph paper, draw $\triangle$HOT whose vertices are H(-02,0). O(0,0), and T(0,4).

    b. Using the same axes, graph $\triangle$H'O'T', the image of $\triangle$HOT under the translation $T_{2,-3}$.

    c. Using the same axes, graph $\triangle$H"O"T", the image of $\triangle$H'O'T' under the translation $T_{1,4}$.

    d. Name a single transformation that is equivalen to $T_{2,-3}$ followed by $T_{1,4}$.

20. a. On graph paper, draw quadrilateral WARM whose vertcies are W(-4,3), A(0,1), R(2,5), and M(o,5).

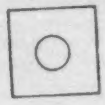
    b. Using the same axes, graph quadrilateral W'A'R'M' where W W' under $T_{s,-2}$.

    c. Using the same axes, graph quadrilateral W"A"R"M" where W" W" under $T_{0,2}$.

    d. Name a single translation that is equivalent to $T_{5, -5}$ followed by $T_{0,2}$.

Rotations Name: _____

In 1-15: A. Tell whether or not the firgure drawn or named has rotational summetry. B. Where possible, find the degree measure of the smallest angle that will rotate the figure to be its own image.

1. 

2.

3.

4.

5.

6. +

7. —

8. Rectangle

9. square

10. X

11. ÷

12. parallelogram

13. circle

14. isosceles triangle

15. equiangular triangle

16. Find the image of the point (-3,0) under the rotation R_{180^0}.

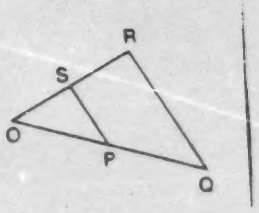
17. What is the image of the point (-1, -4) under a clockwise rotation of 90^0 about the origin, that is, under R_{-90^0}.

In 18, select the numeral preceding the expression that best answers the question.

18. Which geometric figure has 120^0 rotational symmetry? (1) square (2) regular pentagon (3) regular hexagon (4) regular octagon

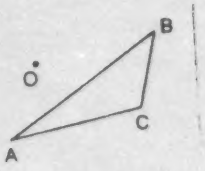
Dilations Name: _____

In 1-5, which refer to the accompanying diagram, O is the center of dilation and $D_k(\triangle OQR) = \triangle OPS$.



1. What is the image of R under the dilation?
2. $D_K(Q) = ?$
3. $D_k(\overline{OR}) = ?$
4. If OP = PQ, what is the constant of dilation k?
5. Using the value of k from exercise 4, find the value of RQ:SP.

In 6-9, copy $\triangle ABC$ and point O, the center of dilation on your paper. Using a ruler, draw its image $\triangle A'B'C'$ under the following conditions:



6. D_2
7. D_3
8. $D_{\frac{1}{2}}$
9. D_{-2}

10. a. On graph paper, draw $\triangle ABC$ whose vertices are A(1,2), B(2,2), and C(2,0).
    b. Using the same axes, graph $\triangle A'B'C'$, the image of $\triangle ABC$ under a dilation of 3.
    c. Using $\triangle ABC$ and $\triangle A'B'C'$, explain why angle measure is preserved.
    d. Using $\overline{BC}$ and $\overline{B'C'}$, explain why midpoints are preserved.
    e. Find the area of $\triangle ABC$ and the area of $\triangle A'B'C'$. Explain why area is <u>not</u> preserved under the dilation.

11. a. On grpah paper, draw $\triangle HOG$ whose vertices are H(1,2), O(0,0), and G(1,3).
    b. Using the same axes, graph $\triangle H'O'G'$ such that d_3 ($\triangle HOG$) = $\triangle H'O'G'$.
    c. Using GH and G'H', show why distance is <u>not</u> preserved under the dilation.
    d. Find the value of each of the following ratios: (1) OG:OG' (2) G'H':GH (3) OH;O'H' (4) OH:HH' (5) G'G:GO

12. The vertices of $\triangle HEN$ are H(0,4), E(6,4), and N(4,-2). Under a dilation of ½, the image of $\triangle HEN$ is $\triangle H'E'N'$.
    a. On one set of axes, graph $\triangle HEN$ and $\triangle H'E'N'$.
    b. Find the lengths of HE and H'E'.
    c. Find the ratio of HE"H'E'.
    d. Find the areas of $\triangle HEN$ and $\triangle H'E'N'$.
    e. Find the ratio of the area of $\triangle HEN$ to the area of $\triangle H'E'N'$.

Line Symmetry Name: _____

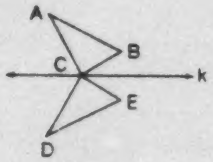
1. On your paper, print all the capital leters of the alphabet that have line symmetry, and draw one or more lines of reflection over these leters.

2. Tell which of the following words has line summetry and, if such symmetry exists, draw a line of reflection through the word.

a. DAD	b. MOM	c. HIKED	d. CHECK
e. BOB	f. DEED	g. RADAR	h. CHOKE
i. AVA	j. TOOT	k. AXIOM	l. YOUTH

3. Using the words from exercise 2 that do not have line symmetry, print the letters of each word in a vertical column. Which of these words now has line symmetry?

In 4-10, which refer to the accompanying diagram, the reflection of △ABC in line K is △DEC.



Ex. 4 to 10

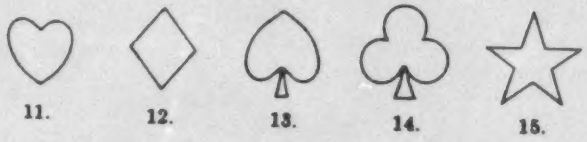
4. What is the image of point A under the line reflection?

5. $r_k(B) = ?$ 6. $r_k(C) = ?$ &. $r_k(D) = ?$

8. What is the preimage of point B under the line reflection?

9. $r_k(\angle ABC) = ?$ 10. $r_k(\overline{DE}) = ?$

In 11-24: a. On your paper, copy the given figure or sketch the geometric figure named. b. Tell the number of lines of symmetry each figure has, if any, and sketch them on your drawing.


11. 12. 13. 14. 15.

16. parabola

17. square 18. rectangle

19. parallelogram 20. rhombus

21. trapezoid 22. line segment

23. circle 24. equilateral triangle

Point Symmetry Name: _____

1. Tell which of the following words has point symmetry and, if such summetry exists, locate the reflection point in the word.

a. SIS b. WOW c. NOON d. ZOO
e. OX f. SWIMS g. un h. pod

2. Which letter has both point symmetry and line symmetry?

(1) S (2) H (3) U (4) N

3. Which letter has point symmetry, but not line symmetry?

(1) C (2) O (3) Z (4) I

4. What kind of symmetry does the letter " X " have?
(1) point symmetry only (2) both point and line symmetry
(3) line symmetry only (4) neither point nor line symmetry

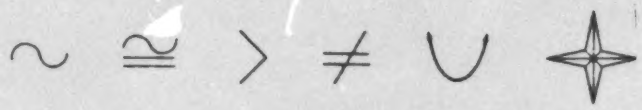
5. Which figure has no point symmetry?

(1) parallelogram (2) square
(3) regular hexagon (4) isosceles triangle

Symmetry Review

Name: _____

In 1-16, for each figure drawn or named: a. Does the figure have line symmetry? If yes, how many lines of symmetry does the figure have? b. Does the figure have point symmetry? c. Does the figure have rotational symmetry? If yes, find the degree measure of the smallest angle that will rotate the figure to be its own image.



1. 2. 3. 4. 5. 6.

7. parallelogram
8. equilateral triangle
9. line segment
10. regular pentagon

11. RADAR

12. BOB

13.

14. DEED

15. CHECK

16.

17. Which letter has no line symmetry?

(1) C (2) A (3) N (4) E

18. Which word has horizontal line symmetry?

(1) NOON (2) CHECKED
(3) MOM (4) CHESS

19. What kind of symmetry does a regular hexagon have?
(1) line symmetry only (2) both line and point symmetry
(3) point symmetry only (4) neither line nor point symmetry

Compositions Name: _____

1. The coordinates of the vertices of $\triangle ABC$ are A(1,3), B(1,6), and C(3,3).
   a. On graph paper, draw and label $\triangle ABC$.
   b. Find the coordinates of the vertices of $\triangle A'B'C'$, the image of $\triangle ABC$ under a point reflection in the origin.
   c. Find the coordinates of $\triangle A"B"C"$, the image of $\triangle A'B'C'$ after a reflection in the y-axis.
   d. Name the single transformation that maps $\triangle ABC$ onto $\triangle A"B"C"$.

2. The vertices of $\triangle ABC$ are : A(-6, -2). B(-4, 1), and C(-1, 4).
   a. Graph $\triangle ABC$ and its image, $\triangle A'B'C'$, under the translation $T_{6,0}$.
   b. Graph $\triangle A"B"C"$, the image of $\triangle A'B'C'$ ater a reflection in the x-axis.
   c. The single transformation that maps $\triangle ABC$ onto $\triangle A"B"C"$ is a:
      (1) line reflection (2) point reflection (3) glide reflection
      (4) rotation

3. The coordinates of the vertices of $\triangle CDE$ are C(0,1), D(4,1), and E(4,3).
   a. Graph $\triangle CDE$ and its image $\triangle C'D'E'$, under a reflection in the x-axis.
   b. Graph $\triangle C"D"E"$, the image of $\triangle C'D'E'$ under a reflection in the line y = x.
   c. The composition $r_{y=x} \circ r_{x\text{-axis}}$ which maps $\triangle CDE$ onto $\triangle C"D"E"$ is equivalent to the transformation:
      (1) $r_{y\text{-axis}}$ (2) $r_{y=-x}$ (3) R_{90^0} (4) R_{270^0}

4. The vertices of $\triangle ACE$ are A(o,2), C(5,4), and E(6,2).
   a. Graph $\triangle ACE$ and $\triangle A'C'E'$ where $R_{90^0}(\triangle ABC) = \triangle A"B"C"$.

      Recall that the rule for a rotation of 90^0 about the origin is $(x,y) \rightarrow (-y,x)$.
   b. Graph $\triangle A"B"C"$ such that $R_{180^0}(\triangle A'B'C') = \triangle A"B"C"$.
   c. Graph $\triangle A"B"C"$ such tat $r_{x\text{-axis}}(\triangle A"B"C") = \triangle A'''B'''C'''$.
   d. The composition $r_{x\text{-axis}} \circ R_{190^0} \circ r_{90^0}$ is equivalent to the sin-

      transformation:
      (1) $r_{y=x}$ (2) $r_{y=-x}$ (3) $r_{y=\text{axis}}$ (4) R_{270^0}

5. The coordinates of the vertices of $\triangle BUG$ are B(1,1), Y(1,4), and G(7, 1). Let $r_{y\text{-axis}}(\triangle BUG) = \triangle B'U'G'$, and $r_{y=x}(\triangle B'U'G') = \triangle B"U"G"$.
   a. on graph paper, draw and lable $\triangle BUG$, $\triangle B'U'G$, and $\triangle B"U"G"$.
   b. State the coordinates of the vertices of $\triangle B'U'G'$.
   c. State the coordiantes of the vertices of $\triangle B"U"G"$.
   d. Name the single transformation that maps $\triangle BUG$ onto $\triangle B"U"G"$.

Composition of 2 Line Reflections Name: _____

In 1-5, select the numeral preceding the expression that best completes the sentence.

1. If line g || line H, then the composition $r_g \circ r_h$ is quivalent to a:
   (1) translation (2) point reflection (3) glide reflection
   (4) line reflection

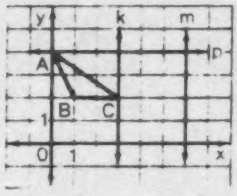
2. If line g $\perp$ line h, then the composition $r_g \circ r_h$ is equivalent to a:
   (1) translation (2) point reflection (3) glide reflection
   (4) line reflection

3. If line g intersects line h, then the composition $r_g \circ r_h$ is equivalent to a:
   (1) translation (2) glide reflection (3) rotation
   (4) line reflection

4. The composition of two line reflections is always:
   (1) a translation (2) a rotation (3) a direct isometry
   (4) an opposite isometry

5. If the composition $r_p \circ r_q$ is equivalent to a rotation of 80^0, then lines p and q intersect to form an angle whose measure is:
   (1) 40^0 (2) 80^0 (3) 90^0 (4) 160^0

In 6-14, copy $\triangle ABC$ and the appropriate lines onto graph paper. The coordinates of the vertices of $\triangle ABC$ are A(0,4), B(1,2), and C(3,2). The equation of line k is x = 3, of line M is x = 6, and of line p is y = 4. Let r_x represent a reflection in the x-axis and let r_y represent a reflection in the y-axis.



6. a. Find the coordinates of the vertices of $\triangle A"B"C"$, the image of $\triangle ABC$ under the composition $r_m \; r_k$.

   b. The composition $r_m \circ r_k$ is equivalent to the isometry:
   (1) $T_{0,6}$ (2) $T_{6,0}$ (3) T_{Q-6} (4) $T_{6,0}$

7. a. Find the coordinates of the vertices of $\triangle A"B"C"$, the image of $\triangle ABC$ under the composition $r_k \; r_m$.

   b. The composition $r_k \circ r_m$ is equivalent to the translation:

   (1) $T_{0,6}$ (2) $T_{6,0}$ (3) $T_{0,-6}$ (4) $T_{-6,\,0}$

8. a. Find the coordinates of the vertices of $\triangle A"B"C"$, the image of $\triangle ABC$ under the composition $r_x \; r_p$.

   b. The composition $r_x \circ r_p$ is equivalent to the translation:

   (1) $T_{0,8}$ (2) $T_{8,0}$ (3) $T_{0,-8}$ (4) $T_{8,0}$

Composition of 2 Line Reflections

9. a. If $r_g \circ r_x$ ($\triangle$ABC) = $\triangle$A"B"C", find the coordinates of A", B', and C".

   b. Name the singe transformation equivalent to $r_p \quad r_x$.

10. a. If $r_x \circ r_y$ ($\triangle$ABC) = $\triangle$A"B"C", find the coordinates of A",B", and C".

    b. Name the singe transformation equivalent to $r_x \circ r_y$.

11. a. If $r_p \quad r_k$ ($\triangle$ABC) = $\triangle$A"B"C", find the coordinates of A", B', and C".

    b. The composition $r_p \circ r_k$ is equivalent to a reflection through the point:

    (1) (3,4) (2) (4,3) (3) (6,4) (4) (0,0)

12. A point reflection through A is equivalent to the composition:

    (1) $r_x \circ r_y$ (2) $r_y \circ r_p$ (3) $r_x \circ r_p$ (4) $r_p \circ r_x$

13. Which composition is <u>not</u> a translation to the right?

    (1) $r_m \quad r_k$ (2) $r_m \quad r_y$ (3) $r_k \quad r_y$ (4) $r_y \quad r_k$

14. a. If $r_p \quad r_m \quad r_k$ ($\triangle$ABC) = $\triangle$A"B"C", find the coordinates of A", B', & C".

    b. The isometry equivalent to $r_p \circ r_m \circ r_k$ is a:
    (1) translation (2) point reflection
    (3) glide reflection (4) line reflection

IMPORTANT RULES TO REMEMBER

Line reflections in the coordinate plane

(1) $(x,y) \xrightarrow{r_{y\text{-axis}}} (-xy)$

(2) $(x,y) \xrightarrow{r_{x\text{-axis}}} (x, -y)$

(3) $(x,y) \xrightarrow{r_{y=x}} (y, x)$

(4) $(x,y) \xrightarrow{r_{y=-x}} (-yx\ -x)$

Rotation rules in the coordinate plane

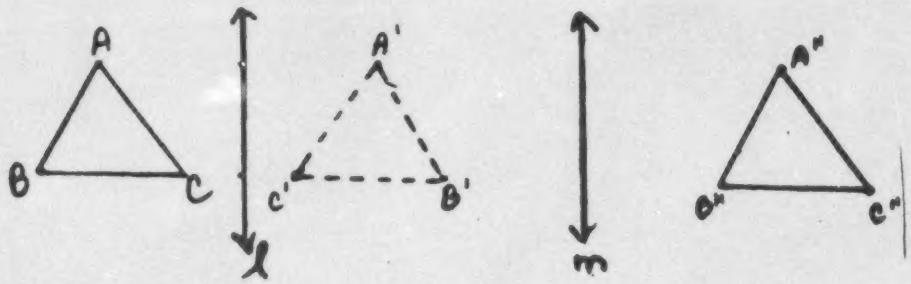
(1) $(x,y) \xrightarrow{R_{90^0}} (-y, x)$

(2) $(x,y) \xrightarrow{R_{-90^0}} (y, -x)$

(4) $(x,y) \xrightarrow{R_{180^0}} (-x, -y)$ [same as reflecting through origin]

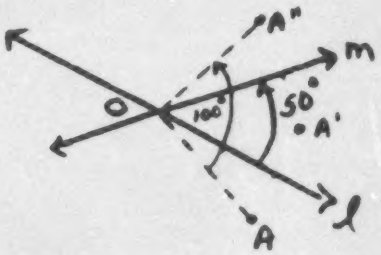
Every translation can be expressed as $r_m \ r_l$ where:

a. $\ell \parallel m$
b. distance from l to m = ½ distance of translation
c. direction of translation $\perp \ell$ and m.



Every rotation $R_{0.2\theta}$ can be expressed as $r_m \circ r_\ell$ where:

a. ℓ and m intersect at O
b. angle formed by ℓ and m in the direction from ℓ to m is θ.



Regents - Transformational Geometry Name:_____

1. Triangle ABC has coordinates A(2,1), B(2,4), and C(4,2). Given the following transformations:

   $T = (x,y) \rightarrow (-x_1 - y)$

   $U = (x,y) \rightarrow (x+4, y-6)$

   $F = (x,y) \rightarrow (3x, 2y)$

   a. Sketch △ABC and its image, △A'B'C' after the transformation T. [3]
   b. Sketch △A"B"C", the image of △ABC after the transformation U. [3]
   c. Sketch △A"'B"'C"', the image of △ABC after transformation F. [2]
   d. Which transformation above is <u>not</u> an isometry? [2]

2. Triange ABC has vertices A(-2,2), B(-7,4), and C(-5,6).

   a. On graph paper, draw and label △ABC. [1]
   b. What are the coordinates of the vertices of △A'B'C', the image of △ABC reflected over the s-axis. [3]
   c. What are the coordinates of the vertices of △A"B"C", the image of △A'B'C' reflected over the line y = -x? [3]
   d. What are the coordinates of the vertices of △A"'B"'c"', the image of △A"B"C" under the translation $T_{(3,-4)}$? [3]

3. a. On graph paper, draw and label △ABC. △ABC has coordinates A(3,1), B(4,3), and C(6.1). [1]
   b. Reflect the figure in part a over the line y = x. What are the coordinates of △A'B'C', the imge of △ABC? [3]
   c. Using the origin as the center of rotation, rotate the figure in part b 90° couterclockwise. What are the coordinates of △A"B"C", the image of △A'B'C'? [3]
   d. Reflect the figure in part c through the origin. What are the coordinates of △A"'B"'C"', the image of △A"B"C"? [3]

4. Triangle ABC has vertices A(9,1), B(2,5), and C(2,1).

   a. On graph paper, draw and label △ABC. [1]
   b. Reflect △ABC over the y-axis. What are the coordinates of the vertices of the image of △A'B'C'? [3]
   c. Reflect △A'B'C' over the line y = x. What are the coordinates of the vertices of this image △A"B"C"? [4]
   d. What is the name of the single transformation which maps △ABC on A"B"C"? [2]

5. Given: F is the transformation $(x,y) \rightarrow (-y,-x)$
U is the transformation $(x,y) \rightarrow (x-2, y+4)$
N is transformation $(x,y) \rightarrow (2x,2y)$

The coordinates of $\triangle$ ABC are A(1,2), B(4,0), and C (3,-2).

a. Sketch $\triangle$ ABC and its image $\triangle$ A'B'C' after the transformation F. [3]
b. Sketch $\triangle$ A"B"C", the image of $\triangle$ A'B'C' after the transformation U. [3]
c. Sketch $\triangle$ A"'B'"C"', the image of $\triangle$ A"B"C" after the transformation N. [3]
d. Which transformation, F, U, or N, is a dilation? [1]

6. Triangle ABC has the coordinates A(2,3), B(9,1), and C(5,5).

a. On graph paper, draw and label triangle ABC. [1]
b. Find the coordinates of A'B'C', the image of ABC after a reflection through the origin. [3]
c. Find the coordinates of the vertices of triangle A"B"C", the image of triangle A'B'C' after $r_{y\text{-axis}}$ or $_{x\text{-axis}}$ ($\triangle$ A'B'C'). [4]

d. A series of transformations which reflect point P through the origin, then over the x-axis, and then over the y-axis, is equivalent to a rotaton about the origin of

(1) 90^0 (3) 270^0
(2) 180^0 (4) 360^0

7. The coordinates of the endpoints of line segment $\overline{AB}$ are A(4,1), and B(5,4).

a. Graph $\overline{AB}$. [2]
b. Graph $\overline{A'B'}$, the image of $\overline{AB}$, after a reflection over the line y = x. [2]
c. Graph $\overline{A"B"}$, the image of $\overline{A'B'}$, after the transformation $(x,y) \rightarrow (x - 5, y - 5)$. [2]
d. Graph $\overline{A"'B"'}$, the image of $\overline{A"B"}$, after a reflection through the origin. [2]
e. Write translation which will map $\overline{A'B'}$ onto $\overline{B"'A"'}$.

Regents - Transformational Geometry Name: _____

1. The capital letter "N" has 1. _____

(1) line symmetry, only
(2) point symmetry, only
(3) both line and point symmetry
(4) neither line nor point symmetry

2. A transformation maps (1,3) onto (-1,-3). This transformation is equivalent to 2. _____

(1) rotation R_{90^0}
(2) rotation R_{-90^0}
(3) dilation D_{-1}
(4) translation $T_{-1,-3}$

3. Which has 60^0 rotational symmetry? 3. _____

(1) regular pentagon
(2) regular hexagon
(3) square
(4) equilateral triangle

4. Which does not have vertical line symmetry? 4. _____

(1) HAH
(2) MUM
(3) BOB
(4) WOW

5. Which transformation is not an example of an isometry? 5. _____

(1) line reflection
(2) rotation
(3) translation
(4) dilation

6. Given: A(0,2) and B(4,0). Which translation maps A A onto the midpoint of AB? 6. _____

(1) $T_{(2,-1)}$
(2) $T_{(-1,2)}$
(3) $T_{(-2,-1)}$
(4) $T_{(-1,-2)}$

7. What kind of symmetry does the letter "H" have? 7. _____

(1) line symmetry, only
(2) point symmetry, only
(3) both line and point symmetry
(4) neither line nor point symmetry

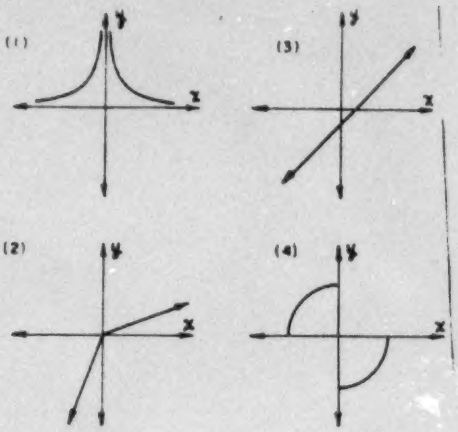
8. Which property is not preserved under a line reflection? 8. _____

(1) collinearity
(2) distance
(3) orientation
(4) perpendicularity

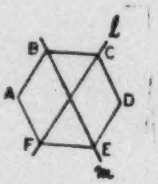
9. If and m are parellel lines, then r r_m (AB) is equivalent to a 9. _____

(1) rotation
(2) dilation
(3) translation
(4) glide-reflection

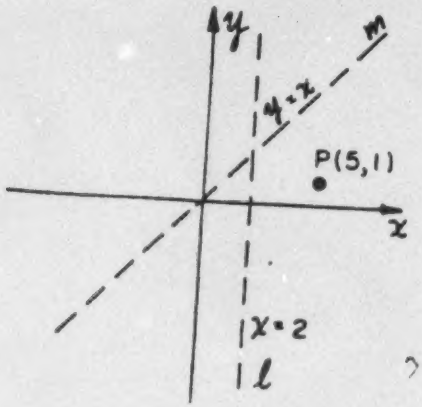
10. Which figures has line symmetry with respect to the line x = 0? 10. _____



11. In the accompanying figure, l and m are symmetry lines for regular hexagon ABCDEF. Find $r_{\ell} \circ r_{m}$ (D). 11. _____



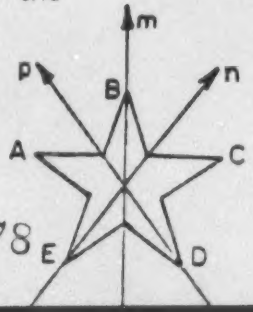
12. In the accompanying diagram, find the coordinates of $r_{\ell} \circ r_{m}$ (P). 12. _____



13. If P (-2,4) is reflected over the line y = -x, what are the coordinates of P', the image of P? 13. _____

14. A translation maps P(4,-4) onto P(3,0). Find the coordinates of Q', the image of Q(3,2), under the same translation. 14. _____

15. In the accompanying figure p, m, and n are symmetry lines. Find $r_{p} \circ r_{m} \circ r_{n}$ (A). 15. _____



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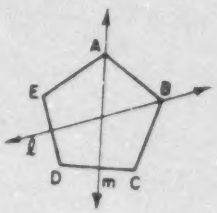
16. A translation maps P(4,-3) onto P(0,0). Find the coordinates of Q', the image of Q(-2,1), under the same translation. 16. _____

17. What is the image of the point (-3, -6) under a rotation of 90^0 counterclockwise about the origin? 17. _____

18. All isosceles trapezoids have 18. _____
(1) point symmetry, only
(2) line symmetry, only
(3) both point and line symmetry
(4) neither point nor line symmetry

19. In the accompanying figure ℓ and m are symmetry lines for regular pentagon ABCDE. 19. _____
What is $r_m \circ r_\ell$ $(\overline{DC})$?



20. If P(3,5) is reflected over the line y = x, what are the coordinates of the image of P? 20. _____

21. a. On graph paper, draw and label $\triangle ABC$ whose coordinates are a(2,1), B(6,4), and C(8,1). [1]
b. Graph and state the coordinates of A'B'C', the reflection of $\triangle$ ABC through the x-axis. [3]
c. Graph and state the coordinates of $\triangle$ A"B"C", the image of $\triangle$ A'B'C' after the translation $T_{-6,-2}$. [3]
d. Using the origin as the center of rotation, graph and state the coordinates of $\triangle$ A"B"C", the result of rotating $\triangle$ A"B"C" 90^0 clockwise.

22. Triangle BC has coordinates of A(1,1), B(3,1), and C(1,4).
a. Graph $\triangle$ ABC and graph and state the coordinates of its image $\triangle$ A'B'C' after the dilation D_2. [3]
b. Graph and state the coordinates of $\triangle$ A"B"C", the image of $\triangle$ A'B'C' after the reflection W: (x,y) (x,-y). [3]
c. Graph and state the coordinates of $\triangle$ A"'B'"C"', the image $\triangle$ A"B"C" after the glide reflection G: (x,y) (y,x - 2). [3]
d. Which of the transformations above D_3 w, or G, is <u>not</u> an isometry? [1]

Regents - Transformational Geometry Name: _____

Multiple Choice

1. Which letter has vertical line symmetry? 1. _____
(1) A (3) N
(2) S (4) D

2. Which property is not preserved under a dilation? 2. _____
(1) distance (3) collinearity
(2) orientation (4) angle measurement

3. Which geometric figure has 120^0 rotational symmetry? 3. _____
(1) square (3) regular pentagon
(2) rhombus (4) equilateral triangle

4. Which graph has line symmetry with respect to the y-axis? 4. _____
(1) $y = x_2$ (3) $y = \sin x$
(2) $y = x^2$ (4) $y = \tan x$

5. Which letter has only point symmetry? 5. _____
(1) A (3) C
(2) H (4) S

6. Which kind of symmetry does a regular hexagon have? 6. _____
(1) line symmetry, only (3) both line and point symmetry
(2) point symmetry, only (4) neither line nor point symmetry

7. Which properties are preserved under a rotation? 7. _____
(1) distance, only
(2) angle measure, only
(3) distance and angle measure, only
(4) distance, angle measure, and orientation

8. If line s intersects line t, then $r_s \circ r_t$ ($\triangle ABC$) is equivalent to a 8. _____
(1) rotation (3) dilation
(2) translation (4) glide-reflection

9. Which word has horizontal line symmetry? 9. _____
(1) ODD (3) COMPOSITE
(2) EVEN (4) PRIME

10. Find the image of (1.5) when it is reflected over the line $y = x$. 10. _____

11. Point P(-1, -5) is reflected over the line $y = -x$. What are the coordinates of P', the image of P? 11. _____

12. A transformation maps P(x,y) onto P(x-2y+3). Under the same transformation, find the coordinates of Q' the image of Q(-4,1). 12. _____

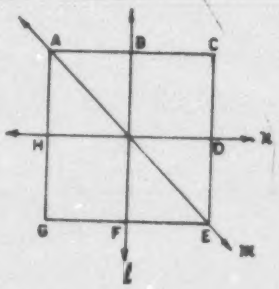
13. What is the image of the point (5,-2) under the translation $T_{2.1}$? 13. _____

14. Find the coordinates of the image of A(-1,2) under the dilation D_4. 14. _____

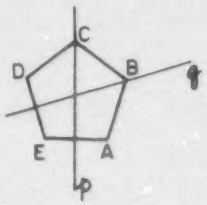
15. Find the image of the point (-3,5) under the rotation R_{180}. 15. _____

16. A translation maps the point (0,0) to a point (-2,4). What is the image of the point (3,-2) under the same translation? 16. _____

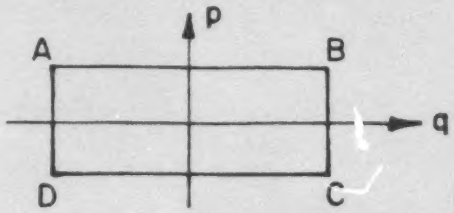
17. In the accompanying figure ℓ, m, and n are symmetry lines. What is $r_{\ell} \circ r_{m}$ ($\overline{CD}$)? 17. _____



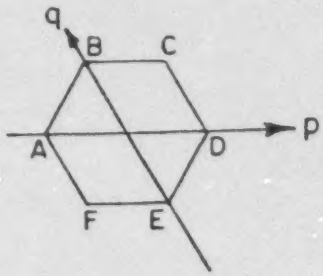
18. In the accompanying figure, pa and q are symmetry lines for regular pentagon ABCDE. Find $r_q \circ r_p$ (B). 18. _____



19. In the accompanying figure, p and q are symmetry lines for rectangle ABCD. Find $r_p \circ r_p \circ r_q$ (A). 19. _____



20. In the accompanying figure, p and q are symmetry lines for regular hexagon ABCDEF. Find $r_q \circ r_p \circ r_q$ (D). 20. _____



21. ABC has coordinates A(6,1), B(8,5), and C(5,4). 21. _____
   a. On graph paper, draw and label $\triangle$ ABC. [1]
   b. Find coordinates of A', B', C', the images of A, B, and C, after a refelction over the line y = x. [3]
   c. Find the coordinates of the vertices of $\triangle$ A"B"C", the image of roiginal ABC after $r_{y\text{-axis}} \circ r_{x\text{-axis}}$ ($\triangle$ ABC). [3]
   d. Find the coordinates of the vertices of $\triangle$ A"'B"'C"', the image of original $\triangle$ ABC after a translation which maps P (0,0) onto P' (3,-1). [3]

22. Triangle ABC has coordinates A(1,-1), B(4,-1) and C(3,-2). Give the following transformations: 22. _____

$T = (x,y) \rightarrow (-x,y)$
$U = (x,y) \rightarrow (x - 4, y + 4)$
$W = (x,y) \rightarrow (2x,2y)$

   a. Sketch $\triangle$ ABC and its image $\triangle$ A'B'C' after the transformation T. [2]
   b. Sketch $\triangle$ A"B"C", the image of $\triangle$ ABC after the transformation u. [3]
   c. Sketch $\triangle$ A"'B"'C"', the image of $\triangle$ ABC after the transformation W. [3]
   d. Which transformation above is <u>not</u> an isometry? [1]
   e. Which transformation above does <u>not</u> preserve orientation? [1]

Regents - Transformational Geometry Naame:_____

1. Which geometric abbreviation has vertical line symmetry? .1. _____
   (1) SSS (3) AAA
   (2) SAS (4) ASA

2. Which is not an isometry? 2. _____
   (1) rotation (3) translation
   (2) reflection (4) dilation

3. Which letter has point symmetry only? 3. _____
   (1) A (3) O
   (2) Z (4) B

4. A dilation is a transformation that does not preserve 4. _____
   (1) distance (3) orientation
   (2) similarity (4) betweenness

5. The graph of which equation has line symmetry with respect to the y-axis? 5. _____
   (1) y = sin x (3) y = xos x
   (2) y = tan x (4) y = x

6. A property not preserved under a line reflection is 6. _____
   (1) angle measure (3) distance
   (2) collinearity (4) orientation

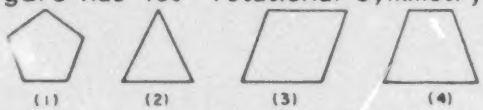
7. Which kind of symmetry does a rhombus have? 7. _____
   (1) line symmetry, only
   (2) point symmetry, only
   (3) both line and point symmetry
   (4) neither line nor point symmetry

8. Which transformation can produce an image triangle with an area not equal to the area of the original triangle? 8. _____
   (1) translation (3) dilation
   (2) line reflection (4) rotation

9. Which letter has both point symmetry and line symmety? 9. _____
   (1) A (3) C
   (2) H (4) S

10. What is the total number of lines of symmetry in any rectangle?
(1) 1 (3) 3
(2) 2 (4) 4 — 10. _____

11. Which figure has 180^0 rotational symmetry? — 11. _____



12. What are the coordinates of the image of point A(-2,-5) after a reflection over the line y = -1? — 12. _____

13. What is the image of the point (2,-3) under a clockwise rotation of 90^0 (r_{-90^0} about the origin? — 13. _____

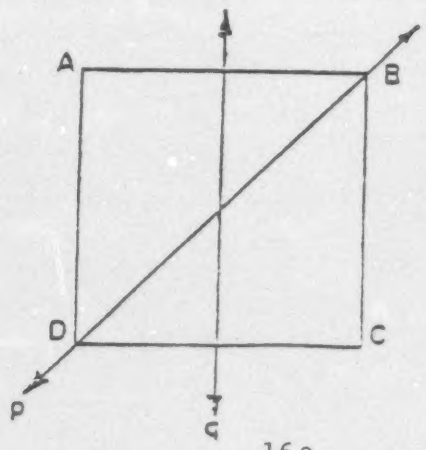
14. If the point A(-5,-1) is reflected about the line y = -x, find the coordinates of the image point A'. — 14. _____

15. A translation maps (5,2) onto (1,-1). What is the image of (0,1) under the same translation? — 15. _____

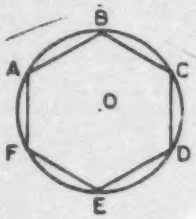
16. A translation maps A(0,2) onto A'(2,0). Find the coordinates of B', the image of B(1,1) under the translation. — 16. _____

17. A translation maps the origin to point (4,3). What is the image of the point (-1,2) under the same translation? — 17. _____

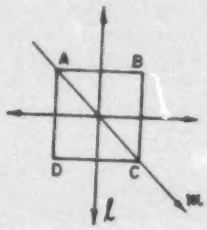
18. In the accompanying figure, pa and q are symmetry lines for square ABCD. Find $r_p \circ r_q \circ r_p$ (C). — 18. _____



19. In the accompanying figure, regular hexagon ABCDEF is inscribed in circle O. With o as the center of rotation, find $R_{-240^0}\ R_{60^0}$ (A). 19. _____



20. In the accompanying figure ℓ and m are symmetry What is $r_{\ell} \circ r_{m}$ (AB)? 20. _____



21. a. On graph paper graph and label triangle ABC whose vertices have coordinates A(4,0), B(8,1), and C(8,4). [1]
    b. Graph and state the coordinates of △A'B'C', the image of △ABC after the composite transformation $r_{x=0} \circ r_{y=x}$ (△ABC). [4]
    c. Which single type of transformation maps △ABC onto △A'B'C'?
    (1) rotation (3) glide reflection
    (2) dilation (4) translation [1]
    d. Graph and state the coordinates of △A"B"C", the image of △ABC after the composite transformation $r_{yx-4} \circ r_{y=0}$ (△ABC). [3]
    e. Which single type of transformation maps △ABC onto △A"B"C"?
    (1) rotation (3) glide reflection
    (2) dilation (4) translation [1]

22. Given A(8,5) and B(6,1) and the transformations T, R, and S as described below:
    T: $(x,y) \rightarrow (x+1y-5)$
    R: $(x,y) \rightarrow (y,x)$
    S: $(x,y) \rightarrow (-x,y)$

    a. Graph $\overline{AB}$ and its image $\overline{A'B'}$ after the transformation T. [3]
    b. Graph $\overline{A"B"}$, the image of $\overline{AB}$ after the transformation R. [2]
    c. Graph $\overline{A'''B'''}$, the image of $\overline{AB}$ after the transformation S [2]
    d. Compare the slopes of the pairs of segments listed below and indicate whether these slopes are euqal, reciprocals, additive inverses, or negative reciprocals.
    (1) $\overline{AB}$ and $\overline{A'B'}$ [1]
    (2) $\overline{AB}$ and $\overline{A"B"}$ [1]
    (3) $\overline{AB}$ and $\overline{A'''B'''}$ [1]

PROBABILITY
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a. Binomial Theorem
b. Probabilities
c. Part II Questions

Permutations Name: _____

1. In how many different orders can you arrange 4 books on the shelf? 1. _____

2. In how many differnet orders can 10 people stand in a line? 2. _____

3. Many radio stations have 4-letter call signs beginning with K. How many such call signs are possible (a) if letters can be repeated? (b) if letters cannot be repeated? 3. _____

4. How many different 3-digit numbers can be formed using the digits 1,2,3,4,5 (a) if digits can be repeated? (b) if digits cannot be repeated? 4. _____

5. In how many ways can 5 people be seated in 10 chairs? 5. _____

6. In how many ways can 4 different prizes be given to any 4 of 9 people if no person receives more than 1 prize? 6. _____

7. a. A high school coach must decide on the batting order for a baseball team of 9 players. Show that there are 362,880 different orders from which the coach can choose. 7.a. _____

   b. Show that there are 40,320 different batting orders with the pitcher batting last. b. _____

8. A track coach must choose a 4-person 400-meter relay team and a 4-person 800-meter relay team from a squad of 7 sprinters, any of whom can run on either team. If the fastest sprinter of the 7 runs last in both races, in how many ways can the coach form the two teams if each of the 6 remaining sprinters runs only once and each different order is counted as a different team? 8. _____

9. How many numbers consisting of 1,2, or 3 digits (without repetitions) can you form using the digits 1,2,3,4,5,6? 9. _____

10. If you have 4 signal flags and can send messages by hoisting one or more flags on a flagpole, how many messages can you send? 10. _____

11. In some states license plates consist of 3 letters followed by 2 or 3 digits (for example, ABC-055 or RRK-54). How many such possibilities are there for those plates with 2 digits? For those with 3 digits? In all, how many license plates are possible. 11. _____

Permutations

12. How many possibilities are there for a license plate with 2 letters and 3 or 4 digits? 12. _____

13. How many 5-digit numbers begin with an odd digit and end with an even digit? 13. _____

14. How many integers between 1000 and 9999, inclusive, contain no 5's? 14. _____

15. In how many different ways can you answer 10 true-false questions? 15. _____

16. In how many different ways can you answer 10 multiple choice questions if each questions has 5 choices? 16. _____

17. a. How many 6-letter "words" can be formed using the letters of the word SENIOR? (NOTE: We allow any arrangement of letters, such as RENISO to count as a "word". We also assume that each letter in the original word is used once.) 17.a. _____

b. How many 5-letter "words" can be formed? b. _____

18. a. How many 5-letter "words" can be formed using the letters of FRESHMAN? 18.a. _____

b. How many of these will have no vowels? b. _____

c. How many will have at least one vowel? c. _____

Combinations Name: _____

1. a. In how many ways can a club of 20 members choose a president and a vice-president? 1.a. _____
   b. In how many ways can they choose a 2-person committee? b. _____

2. a. In how many ways can a club of 13 members choose 4 different officers? 2.a. _____
   b. In how many ways can they choose a 4-person committee? b. _____

3. A couple has room for 5 guests at their dining table. In how may ways can they choose 5 dinner guests from a group of 10 friends? 3. _____

4. A teacher has a collection of 20 true-false questions and wishes to choose 5 of them for a quiz. How many quizzes can be made if the order of the questions is considered (a) important, (b) unimportant? 4. _____

5. a. A hiker wishes to invite 7 firends to go on a trip but only has room for 4 of them. In how many ways can she chooe the 4? 5.a. _____
   b. If she only had room for 3 friends, in how many ways could she choose the 3? How is your answer related to the answer for part (a)? Why? b. _____

6. a. In how many ways can you choose 3 letters from the word LOGARITHM if you are not concerned with the order of the letters? 6.a. _____
   b. In how many ways can you choose 6 letters from LOGARITHM if the order of letters is unimportant? Compare your answer with part (a). b. _____

7. How many different ways are there to deal a hand of 5 cards from a deck of 52 cards? 7. _____

8. Eight people apply for 3 job openings. In how many ways can the 3 openings be filled if the jobs are (a) all different? (b) all the same kind? 8. _____

9. A town council consists of a mayor and 8 aldermen.
   a. How many different committees of 4 can be chosen from this council? 9.a. _____
   b. How many of these committees will have the mayor on it? b. _____
   c. How many will not have the mayor on them? c. _____
   d. Verify that the answer to part (a) = answer to part (b) + answer to part (c).

10. If you have a $1 bill, a $5 abill, and a $20 bill, many different sums of money can you make using one or more of these bills. 10. _____

11. Pat's Pizza Place offers pepperoni, mushrooms, sausages, onions, anchovies, and peppers as toppings for their regular plain pizza. How many different pizza can be made? 11. _____

Circular Combinations Name: _____

1. In how many ways may 6 children form a ring? 1. _____

2. In how many ways may 5 men sit at a round table? 2. _____

3. In how many ways can 7 men be seated at a round table? 3. _____

4. In how many different orders may 10 people form a ring? 4. _____

5. In how many orders may 4 men and 4 ladies be seated at a round table so that the men and ladies alternate? 5. _____

6. Six men and six ladies are to be seated at a round table. In how many different ways may they be seated in order that there shall be a man at the right of each lady? 6. _____

7. Four men and 4 women are seated at random at random at a round table. What is the probability that the men and women will alternate? 7. _____

Probabilities Name: _____

1. If a card is draw from a well-shuffled deck of 52 cards, what is the probability that it is:
   a. a spade? b. a black card? c. a jack, queeen, or king? 1. _____

2. One of the 10 positive integers is picked at random. What is the probability that it is:
   a. even? b. divisible by 3? c. a prime? (Note: 1 is <u>not</u> a prime.) 2. _____

3. Two dice are rolled. What is the probability that their sum is:
   a. 6? b. 7? c. 8? 3. _____

4. Two different numbers are picked from the numbers 1,2,3, and 4. What is the probability that their sum is greater than 4? 4. _____

5. Two cards are dealt from a standard deck. What is the probability that both are:
   a. hearts? b. red cards? 5. _____

6. Two cards are dealt from a standard deck. What is the probability that both are:
   a. aces? b. black aces? 6. _____

In Exercises 7 and 8 leave your answers in terms of factorials unless directed otherwise by your teacher.

7. Thirteen cards are dealt from a standard deck. What is the probability of getting:
   a. all clubs? b. no clubs? 7. _____

8. Five cards are dealt from a standard deck. What is the probability of getting:
   a. all four aces? b. no aces? 8. _____

9. There are 5 red balls and 3 white balls in a bag. If 2 balls are drawn at random, what is the probability that the number of red balls is 0? 1? 2? 9. _____

10. Repeat Exercise 9 if the bag contains 6 red balls and 2 white balls? 10. _____

11. Suppose you shuffle together the 12 face cards from a deck (that is, the jacks, queens, and kings). If 3 of these 12 cards are drawn, find the probability that each of them is a:
    a. jack b. spade c. black card 11. _____

12. Suppose a die is rolled twice and comes up "3" on the first roll. What is the probability that the second roll also shows "3"? 12. _____

13. Suppose someone rolls two dice and announces that their sum is 6. What is the probability that both dice show "3"? 13. _____

14. If the probability of getting a sum of 10 when a die is rolled twice is 1/12. what is the probability of <u>not getting</u> a sum of 10 when a die is rolled twice? 14. _____

15. A coin is tossed 9 times and comes up "heads" each time. What is the probability that it will come up "heads" on the next toss? 15. _____

At Least - At Most
(Bernoulli)

Nme:_____ _____

1. A fair coin is tossed 4 times. Find the probability of tossing:
   a. exactly 3 heads — a. _____
   b. exactly 4 heads — b. _____
   c. at least 3 heads — c. _____
   d. at most 1 head — d. _____

2. If a fair coin is tossed 6 times, find the probability of obtaining at most 2 heads. 2. _____

3. If 5 fair coins are tossed, what is the probability that at least 2 are tails? 3. _____

4. A fair die is rolled 3 times. Find the probability of rolling:
   a. exactly 1 five — b. no fives — a._____ b._____
   c. at most 1 five — d. at least 2 sixes — c._____ d._____
   e. at most 2 fours — f. exactly 1 even number — e._____ f._____
   g. at most 1 even — h. at most 1 number less than three — g._____ h._____
   i. at least 2 even — j. at least 1 number greater than four — i._____ j._____

5. If a fair die is tossed 5 times, find the probability of obtaining at most 2 ones. 5. _____

6. If 4 fair dice are rolled, what is the probability that at least 2 are fives? 6. _____

7. A coin is weighted so that the probability of heads is 4/5.
   a. What is the probability of getting at least 3 heads when the weighted coin is tossed 4 times? — a._____
   b. What is the probability of getting at most 2 tails when the weighted coin is tossed 5 times. — b._____

8. Each evening the members of the Sanchez family are equally likely to watch the news on any one of 3 possible TV channels: 5,8, and 13.
   a. What is the probability that they watch the news on channel 8 on at least 3 out of 5 evenings? — a._____
   b. What is the probability that they watch the news on channel 13 on at most 2 out of 5 evenings? — b._____

9. In a game, the probability of winning is 1/5 and the probability of losing is 4/5. If 3 games are played, what is the probability that the team will win at least 3 of the next 4 games? 9. _____

10. In each game that the school basketball team plays, the probability that the team will win is 2/3. What is the probability tht the team will win at least 3 of the next 4 games? 10. _____

11. A die is loaded so that the probability of rolling a one is 3/4. What is the probability of rolling at least 2 ones when the die is tossed 3 times? 11. _____

12. If a family of 4 children is selected at random, what is the probability that at most 3 of the children are boys? 12. _____

At Least - At Most
(Bernoulli)

In Exercises 13-14 an electronic game contains 9 keys. As shown below, 5 keys have numbers, and 4 keys have colors. Each key is equally likely to be pressed on each move.

Red	1	Blue
2	3	4
Green	5	Yellow

13. If one key is pressed at random in the electronic game, find: a.__ __ b.____
    a. P(number key) b. P(color key)

14. If 3 keys are pressed at random in the electronic game, find the probability of selecting:
    a. exactly 1 color key b. at least 2 color keys a.____ b.____
    c. at most 1 color key d. all color keys c.____ d.____
    e. exactly 1 number key f. at least 1 number key e.____ f.____

Binomical Theorem Name: _____

In Exercises 1-10 give the series expansion of the given expression. Simplify your answers.

1. a. $(a + b)^6$ b. $(a - b)^6$

2. a. $(X + y)^7$ b. $(x - y)^7$

3. a. $(p + q)^5$ b. $(p - q)^6$ c. $(2p - q)^5$

4. a. $(a + b)^3$ b. $(x - 3y)^3$ c. $(x^2 - 3y)^3$

5. $(1 + a/2)^4$ 6. $(2x^2 - 1)^5$ 7. $(s^2 + 1)^6$

8. $(a^2 - b^2)^4$ 9. $(20 + 1)^3$ 10. $(20 - 1)^3$

In Exercises 11-14 find the first four terms of the series expansion of the given expression. Do <u>not</u> simplify your answers.

11. $(a^2 - b)^{100}$ 12. $(3p + 2q)^{20}$

13. $(\sin x + \sin y)^{10}$ 14. $(\sin x - \cos y)^{30}$

15. In the expansion of $(a + b)^{12}$, what is the coefficient of a^8b^4? of a^4b^8?

16. In the expansion of $(a + b)^{20}$, what is the coefficient of $a^{17}b^3$? of a^3b^{17}?

Regents - Binomial Theorem Name: _____

1. Find the third term of the expansion of $(x - 2)^6$.

Multiple Choice:

2. The middle term in the expansion $(x + 3y)^4$ is: 2. _____

(1) $54x^2y^2$ (3) $12x^3y$

(2) $8x^2y^2$ (4) $12xy^3$

3. What is the last term in the expansion $(2x - 3y)^4$? 3. _____

(1) $-81y^4$ (2) $3y^4$

(2) $81y^4$ (4) $-54xy^3$

4. The third term of the expansion of $3x - y)^4$ is: 4. _____

(1) $54x^2y^2$ (3) $12xy^3$

(2) $-54x^2y^2$ (4) $-12xy^3$

5. What is the third term in the expansion of $(a - 3b)^5$? 5. _____

(1) $90a^3b^2$ (3) $-45a^2b^2$

(2) $45a^3b^2$ (4) $-90a^3b^2$

6. Which is the 7th term in the expansion of $(2x - y)^7$? 6. _____

(1)) $7xy^6$ (3) $14xy^6$

(2) $-7xy^6$ (4) $-14xy^6$

7. What is the numerical coefficient of the second term of the expansion $(x - 2y)^4$? 7. _____

(1) -8 (3) 24
(2) 8 (4) -32

8. Which is the fourth term in the expansion of $(x + 3)^5$? 8. _____

(1) $270x^2$ (3) $+405x$

(2) $135x^2$ (4) $+135x$

9. Which is the third term in the expansion of $(x - 3)^5$? 9. _____

(1) $-9x^3$ (3) $-90x^3$

(2) $9x^3$ (4) $90x^3$

10. What is the third term in the expansion $(x - 2y)^8$? 10. _____

(1) $20x^3y^2$ (3) $80x^2y^3$

(2) $40x^3y^2$ (4) $-80x^2y^3$

11. Which is the 3rd term in the expansion of $(2x - y)^4$? 11. _____

(1) $-32x^3y$ (3) $24x^2y^2$

(2) $12x^2y^2$ (4) $-8xy^3$

Regents - Probabilities Name: _____

1. When a certain machine makes parts, 10 percent of them are defective. In a sample of 10 parts, what is the probability that 3 will be defective?

   (1) $10C3\ (1/10)^7\ (9/10)^3$ (3) $(9/10)^7\ (1/10)^3$

   (2) $10C3\ (9/10)^7\ (1/10)^3$ (4) $(1/10)^7\ (9/10)^3$

2. Three fair coins are tossed. What is the probability of obtaining <u>at least</u> two tails?

3. Three fair coins are tossed. What is the probability of obtaining <u>at most</u> two heads?

4. If a fir coin is tossed 4 times, find the probability of obtaining <u>at least</u> 3 heads?

5. A coin is tossed 4 times. What is the probability of getting exactly 3 heads?

6. In a game, the probability of winning is 1/3 and of losing is 2/3. If three games are played, what is the probability of winning <u>at least</u> two games?

7. In each game of a baseball series, a team's probability of winning is 1/4 and its probability of losing is 3/4. What is the team's probability of winning at least 2 out of 3 games?

8. Team A and team B play 4 games. If team A's probability of winning a game is 2/5, what is the probability of team A winning at least two games?

9. A fair die is tossed three times. The probability of obtaining exactly 2 fives is _____?

10. The probability of answering correctly by guessing on a multiple choice question is 1/3. What is the probability of guessing correctly three or more questions out of four?

11. Given the set of functions: (sin x, cos x, tan x). What is the probability that a function chosen at random from this set is positive in both <u>Quadrants I and II</u>?

12. If the probability of winning a game is 1/4, find the probability of winning at <u>least</u> 3 games out of 4?

13. From the letters of the word "MATH", a letter is drawn at random and replaced. What is the probability of picking exactly two A's in three tries?

14. The probability of guessing the correct answer on a true-false question is 1/2. If Mary guesses the answers to five true-falso questions, what is the probability that she will get exactly one wrong answer?

15. If the probability of a team winning is 2/3, and the probability of losing is 1/3, what is the probability that the team will win exactly 1 of 4 games?

Probabilities - Part II Questions Name: _____

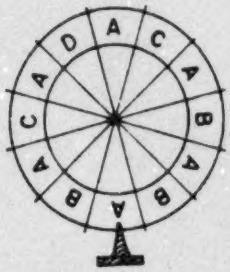
1. A coin is loaded so that the probability of obtaining a head is twice as great as that of obtaining a tail.

   a. Find the value of P_{heads} and P_{tails} on a single toss. [2]

   b. Using the probabilities found in part a, find the probability of:

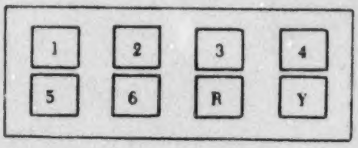
      1. getting exactly 3 heads in 4 tosses [3]
      2. getting all tails in 5 tosses [2]
      3. getting at least 2 heads in 3 tosses [3]

2. The wheel shown in the accompanying diagram is divided into twelve regions of equal area labeled as shown. On any spin of the wheel, each of the regions is equally likely to stop at the arrow.



   a. Find:
      (1) P(A) [1]
      (2) P(B) [1]
   b. Find the probability of:
      (1) getting at least three A's in four spins of the wheel [4]
      (2) getting no more than one B in three spins of the wheel [4]

3. A new electronic game has the form of a calculator as hown. There are eight keys: six of them are numbered 1,2,3,4,5,6 and two are colored, one red: (R), and one yellow (Y). Each key is equally likely to be pressed.

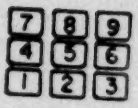


   a. If one key is pressed at random,
      (1) what is the probability that a number key will be selected? [2]
      (2) what is the probability that a color key will be selected? [2]
   b. If three keys are pressed at random,
      (1) what is the probability that exactly two number keys will be selcted? [3]
      (2) what is the probability that at least two number keys will be selcted? [3]

4. The receivable channels on a TV are indicated on the channel selector shown. The probability of selecting each channel is the same.

   a. Find:
      (1) P(2) [1]
      (2) P(even channel) [1]
      (3) P(odd channel) [1]
   b. Find the probability of:
      (1) chossing exactly two even channels on three random selections [3]
      (2) Choosing at least two odd channels on three random selections [4]

5. The numeric key pad on a calculator is arranged as shown in the diagram. The probability of pressing any key at random is the same for each key.



a. Find:

(1) P(6) [1]
(2) P(even number) [1]
(3) P(odd number) [1[

b. Find the probability of:
(1) pressing exactly 2 even numbers on three random presses [3]
(2) getting at least 2 even numbers on random presses [4]

STATISTICS
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SIGMA NOTATION Name: _____

In 1-18, find the value indicated by the summation symbol.

1. $\sum_{k=0}^{6} 2b$
2. $\sum_{i=1}^{5} (i + 1)$
3. $\sum_{j=0}^{4} j^2$
4. $\sum_{n=2}^{5} (n - 2)^2$
5. $\sum_{j=1}^{3} j^3$
6. $\sum_{n=0}^{3} 2^n$
7. $\sum_{n=1}^{4} 1/n$
8. $\sum_{k=0}^{5} (10-k)$
9. $\sum_{i=3}^{7} (2 - 1)^2$
10. $5\sum_{n=1}^{4} (n - 1)$
11. $\sum_{k=1}^{3} 3k^2$
12. $\sum_{k=1}^{3} k^{k-1}$
13. $\sum_{n=0}^{2} \cos(n\pi)$
14. $\sum_{n=1}^{3} (\sin(n\pi/s)$
15. $\sum_{k=0}^{4} \cos(k\pi/2)$
16. $3\sum_{k=1}^{5} (k - 1)$
17. $\sum_{n=1}^{3} (\text{Arc sin } n/2)$
18. $\sum_{n=1}^{3} \tan \frac{(2n - 1)}{4}$

19. The sum 1 + 8 + 27 is not equal to:

(1) $\sum_{i=1}^{4} i^3$ (2) $\sum_{k=0}^{4} (k + 1)^3$

(3) $\sum_{n=0}^{3} (n + 1)^3$ (4) $\sum_{j=2}^{5} (j - 1)^3$

20. Rerpesent the sum 5 + 10 + 15 + 20 + 25 + 30 by three different expressions, each involving the summation sumbol.

In a-d, evaluate the expression:

a. $\sum (3n + 2)$ b. $3\sum_{n=4}^{7} (n + 2)$

c $\sum (3n + 6)$ d. $3\sum_{n=4}^{7} (n + 6)$

e. Which expressions in this exercise, if any, represent the same sum?

22. Show that $\sum_{k=1}^{n} 7k = 7\sum_{k=1}^{n} k$.

23. Show that $\sum_{k=1}^{n} (k - 1) = \sum_{k=2}^{n-1} k$.

Standard Deviation
Frequency of 2

Name: _____

1. For the data given in the accompanying table.
   a. Find the mean.
   b. Copy and complete the table.
   c. Find the standard deviation to the nearest tenth.

x_i	$\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
30			
29			
26			
23			
12			

2. The scores of six students on an IQ test are listed below.

Fred:	130	Toni:	127	Lee:	125
Paul:	122	Lynn:	128	John:	118

For these scores, find: a. the mean b. the standard deviation.

3. The highest number of points that a student can score in a mathematics competition is five. In the last six competitions, Jennifer's scores were: 2, 1, 4, 2, 4, 5. For these scores, calculate: a. the mean b. the standard deviation to the nearest tenth.

4. On his last five fishing trips, Jim caught the following number of fish: 6, 5, 12, 3, 9. For ths data, find: a. the mean b. the standard deviation to the nearest tenth.

5. On a test, five students received scores of 63, 60, 59, 57, and 56. For these scores, find: a. the mean b. the standard deviation to the nearest tenth.

6. The heights in centimeters of five players on the basketball team are listed at the right. For these heights, find: a. the mean b. the standard deviation to the nearest tenth.

Player	Height (cm)
R. Melendy	192
C. Cronin	189
D. Schmeling	187
M. Natale	184
R. Weinrich	183

7. Two students were comparing their grade on their report cards.

Sean O'Brien:	83, 92, 79, 65, 82, 85
John Parisi:	83, 75, 78, 86, 77, 87

a. Which student, if either, has the higher mean average? b. For Sean's grades, calculate the standard deviation to the nearest tenth. c. For John's grades, calculate the standard deviation to the nearest tenth. d. Which student has the greater dispersion in grades?

8. During a 5-day work week, Helen worked the following number of hours per day: 8, 10, 9, 11, 10. For this data, calculate: a. the mean b. the standard deviation to the nearest tenth.

9. Dave worked the following number of hours at a fast-food restaurant over a period of 7 days: 5, 4, 0, 9, 7, 0, 3. Using n = 7, find a. the mean number of hours worked per day. b. the standard deviation to the nearest tenth.

10. Over the last 7 days, Mr. Kavanagh spent the following number of hours reading a novel: 3, 1, 5, 2, 1.5, 4, 2.5, and 3. For these times, find: a. the mean b. the standard deviation to the nearest tenth.

Standard Deviation
Grouped Data

Name: _____

1. The set of data 12, 15, 12, 10, 9, 12, 9, 10, 12, 9 has been organized into a table that follows. for this grouped data: a. Find the mean. b. Copy and complete the table. c. Find the standard deviation to the nearest tenth.

Measure (x_i)	Frequency (f_i)	$f_i x_i$	$x_i - x$	$(x_i - x)^2$	$f_i(x_i - x^2$
15	1				
12	4				
10	2				
9	3				

In 2-4, for the grouped data, find: a. the mean b. the standard deviation to the nearest tenth.

2.

Measure x_i	Frequency f_i
31	1
25	5
22	2
20	2

3.

Measure x_i	Frequency f_i
40	4
30	10
20	4
10	2

4.

Measure x_i	Frequency f_i
10	4
8	2
7	2
5	7

5. The high school juniors were chosen at random. Their non-verbal scores on the PSAT are: 68, 65, 70, 70, 68, 58, 65, 68, 70, and 68. Find the standard deviation of these scores to the nearest tenth.

6. Mr. McEntee is a guidance counselor. During the last 8 school days, he saw the following number of students: 12, 8, 10, 12, 15, 10, 10, and 3. For this data, find the standard deviation to the nearest tenth.

7. Ten Speedo cars were tested at random. These cars were found to average the following number of miles per gallon: 30, 31, 31, 29, 32, 30, 23, 31, 32, and 31. A. Find the mean for this data. b. Find the standard deviation ot the nearest tenth. c. Mr. Pappas owns the car that averages 23 mpg. Does his car have a rate that lies within two standard deviations of this mean?

8. The ages of 10 teachers in the mathematics department are: 38, 41, 34, 28, 41, 41, 24, 38, 41, and 34. For this data, find: a. the mean b. the standard deviation to the nearest tenth c. the mean absolute deviation to the nearest tenth.

Standard Deviation
Grouped Data

9. Mrs. Barash analyzed a test given to 30 students in her class. Using the accompanying table, find:
a. the mean score b. the standard deviation to the nearest tenth.

Interval	Midpoint x_i	Frequency f_i
94-100	97	2
87-93	90	5
80-86	83	15
73-79	76	7
66-72	69	1

10. The test scores of 50 students are grouped into intervals as shown n the table at the right. For this set of grouped data, find:
a. the mean b. the standard deviation.

Interval	Frequency
96-100	2
91-95	8
86-90	8
81-85	5
76-80	11
71-75	10
66-70	5
61-65	1

Normal Distribution Name: _____

In 1-4, write the number that makes the sentence true.

1. In a normal distribution, the mean is at the _____ percentile.

2. Approximately _____% of the scores in a normal distribution lie within one standard deviaton of the mean.

3. A core that is one standard deviation below the mean in a normal distribution is at the _____ percentile.

4. In a normal distribution, about _____% of the scores lie in the interval from $x - 2s$ to $x + 2s$.

5. On a standardized test, the mean is 82 and the standrad deviation is 6. Find th scores for: a. $x + 2s$ b. $x + s$ c. $x - s$ d. $x - 2s$

6. On a standardized test, the mean is 70 and the standard deviation is 10. If Rita Keane scored 90 on this test, then her score is at the:
(1) 84th percentile (2) 90th percentile (3) 95th percentile
(4) 97.5 percentile

7. If the mean score on an IQ test is 104 and the standard deviation is 7, which score will occur less than 5% of the time? (1) 93 (2) 97
(3) 116 (4) 119

8. Mr. Noren bowls regularly and has an average of 182. His bowling scores approximate a normal distribution with a standard deviation of 8.5 He can expect to bowl a core of 200 or better: (1) about 16% of the time (2) between 10% and 15% of the time (3) between 5% and 10% of the time (4) less than 3% of the time

9. The times of telephone calls approximate a normal distribution. If the mean time is 4.3 minutes, with a standard deviation of 1.4 minutes, which length call should occur less than 5 times out of 100?
(1) 2.1 minutes (2) 7.4 minutes (3) 3 minutes (4) 6 minutes

10. For a certain population, the mean score on the Scholastic Aptitude Test (SAT) is 430, with a standard deviation of 108.
    a. Which of the following scores should occur the greatest number of times? (1) 301 (2) 335 (3) 539 (4) 550
    b. Which of the following scores might represent the 70th percentile? (1) 505 (2) 540 (3) 590 (4) 700
    c. A student with a score of 322 does as well as or better than what percent of the population? (1) 2.5% (2) 16% (3) 30% (4) 32.2%

11. Ms. Surber owns a real-estate business. Sale preices of homes in her area approximate a normal distribution, with a mean of $72,000 and a standard deviation of $7,600. A home that sells for $87,600 would rank: (1) below the 75th percentile (2) between the 75th and 85th percentile (3) between 85th and 95th percentiles (4) above the 95th percentile

12. In a large school district, the years of service for the teaching staff approximate a normal distribtution. The mean is 16 years of teaching, and the standard deviation is 7.4 years. Four teachers, and the nu,-ber of years of service for each are listed below.

Robert Novak:	16 years	Mabel Oestrich:	35 years
Samuel Backer:	21 years	Nancy Garbowski:	12 years

According to years of service, name the teacher or teachers of those listed (if any) who might possibly rank at the:

a. 10th percentile b. 35th percentile c. 50th percentile
d. 70th percentile e. 90th percentile f. 99th percentile

In 13-16, assume a normal distribution. Using the given information, find:
a. the standard deviation b. the mean

13. $\bar{x} - s = 75$ and $\bar{x} + s = 109$

14. $\bar{x} - s = 4.5$ and $\bar{x} + 2s = 25.5$

15. Scores within one standard deviation of the mean range from 9.2 to 14.

16. About 2.5% of the scores are at or below 45, and 2.5% are above 77.

Review Name: _____

In 1-4, find the value indicated by the summation sumbol.

1. $\sum_{n=1}^{3} n^n$ 2. $\sum_{j=2}^{4} (5 - i)^2$ 3. $\sum_{k=0}^{4} 3(k = 1)$ 4. $\sum_{n=0}^{2} \sin(n\pi)$

5. Which statistical term represents the score that occurs most often in a distribution? (1) mean (2) median (3) mode (4) standard deviation

6. If Sal's test grades are 73, 90, 87, 98, and 92, what is the range of his grade?

7. The hours that Carol worked over the last 8 weeks are recorded in the table at the right. For this data, find: a. the mean b. the median c. the mode

Measure x_i	Frequency f_i
34	1
32	4
30	1
28	2

Index i	Measure x_i	Frequency f_i
1	40	4
2	25	2
3	20	3
4	10	1

8. For the grouped data in the accompanying table, which is true?
(1) The mean equals the median.
(2) The mean exceeds the median by 3.
(3) The median exceeds the mean by 3.
(4) The mean equals the mode.

9. In a calss, 15 students are 17 years old and 10 students are 16 years old. The mean age of these students is: (1) 16.3 (2) 16.5 (3) 16.6 (4) 16.8

10. Two players at a mini-golf course had the same average for the first nine holes of golf.

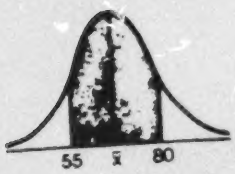
Len: 3, 2, 5, 3, 3, 1, 5, 3, 2 (mean = 3)
Stan: 3, 2, 5, 3, 3, 3, 2, 3, 3 (mean = 3)

a. Find the mean absolute deviation for each player, espressed as a fraction. b. Which player has more widely dispersed scores?

11. The weights of a group of people approximate a normal distribution. If the mean witght is 64 kg and the standard deviation is 4.8, which weight is expected to occur less than 3% of the time? (1) 56 (2) 59.3 (3) 69.8 (4) 74

12. On a standardized exam, the mean is 83 and the standard deviation is 5.5. Which score might be assigned to a student at the 70th percentile? (1) 70 (2) 77.5 (3) 86 (4) 90

13. In the diagram, about 68% of the scores fall into the shaded area. If the distribution is normal and the scores range from 55 to 80, find: a. the standard deviation b. the mean c. the score at $\bar{x} + s_x$.



Ex. 13

14. Over the last 5 days, Mrs. Tountas received the following numbers of letters in the mail: 7, 4, 9, 2, 3. For this data, find: a. the mean b. the standard deviation to the nearest tenth.

15. The scores earned by 5 students on the PSAT are 62, 57, 55, 51, and 50. Calculate the standard deviation of these scores to the nearest tenth.

16. For the grouped data in the accompanying table, find:
a. the mode b. the median
c. the mean d. the standard deviation

Measure (x_i)	Frequency (f_i)
60	1
55	3
50	3
45	5

17. Doris scored the following grades on her last 10 science tests: 85, 100, 90, 90, 100, 85, 80, 90, 100, and 90. Organize these grades into a table of grouped data, and find: a. the mean b. the standard deviation to the nearest tenth.

Regents Review - Sigma Notation

Name: _____

1. Find $\sum_{k=1}^{3} k^k$ 1. _____

2. Find: $\sum_{K=1}^{3} (3 - k)^2$ 2. _____

3. Find the value of $\sum (n^2 + n)$. 3. _____

4. Find: $\sum_{k=1}^{3} 2(k + 1)$

5. Find: $\sum_{k=1}^{4} k^2$ 5. _____

6. Find the value of $\sum_{n=1}^{3} (2n - 1)$ 6. _____

7. Evaluate: $\sum_{k=1}^{3} k^3$ 7. _____

8. Find the value of $\sum_{k=1}^{2} \sin \frac{k}{2}$ 8. _____

9. Evaluate: $\sum_{n=2}^{5} (2n + 3)$ 9. _____

10. Find: $\sum_{n=3}^{5} 3n + 2$ 10. _____

11. Evaluate: $\sum_{k=2}^{4} (k^2 - 1)$

12. Evaluate: $\sum_{k=5}^{7} (k - 2)^2$ 12. _____

Regents Review - Standard Deviation Name: _____

1. Given the following ten scores: 61, 65, 65, 65, 57, 59, 71, 74, 75, 78
For these scores, find the:
   a. mode [1]
   b. median [1]
   c. mean [2]
   d. standard deviation to the nearest tenth [6]

2. A high school football team scored the following number of points during the ten-game season: 19, 20, 21, 27, 29, 29, 34, 40, 40, 41
   a. What is the median? [1]
   b. What is the mean? [2]
   c. Find the standard deviation of these scores to the nearest tenth.

3. Find, to the nearest tenth, the standard deviation for the following set of data. [10]

x_i measure	f_i frequency
60	1
75	4
80	3
90	2

4. The table shows the heights of 10 students taken at random in a junior high school. Determine the standard deviation of their heights to the nearest tenth. [10]

index (i)	measure of height (x_i)	frequency (f_i)
1	56	1
2	58	2
3	60	2
4	61	2
5	62	3

5. The table below shows the weights of ten girls from a ninth grade class. Determine the standard deviation of these weights to the nearest tenth. [10]

Measure of Weight (x_i)	Frequency (f_i)
91	1
96	1
105	2
111	3
113	2
114	1

6. Find, to the nearest tenth, the standard deviation for the following set of data:
   95, 85, 90, 80, 85, 85, 80, 95, 85, 80 [10]

7. On the PSAT, five students received scores of 50, 52, 53, 59, and 61. Calculate the standard deviation of these scores to the nearest tenth. [5,5]

8. On a math test, five students received the following scores: 92, 86, 98, 88, 96. Calculate the standard deviation of these scores to the nearest tenth. [5,5]

9. On a math test, five students received the following scores: 96, 84, 100, 88, 82. Calculate the standard deviation of these scores to the nearest tenth. [5,5]

10. During a given week, Pat worked 7 hours on Monday, 10 hours on Tuesday, 7 hours on Wednesday, 8 hours on Thursday, and 10 hours on Friday. Calcualte the standard deviation of the number of hours worked to the nearest tenth. [10]

11. The ages of ten teachers at George Washington Elementary School are 33, 23, 36, 29, 36, 36, 33, 29, 36, and 29. Determine the standard deviation of these ages to the nearest tenth. [10]

12. The shoe sizes of the ten players on a basketball team are as follows: 7, 8, 9½, 10, 10, 10, 10½, 11, 11, 13. Determine the standard deviation of these shoe sizes to the nearest tenth. [10]

13. The heights in inches of a basketball team are as follows: 81, 73, 75, 80, 71. Calculate the standard deviation of these heights to the nearest tenth. [10]

 Name: _____

1. Given the following table of SAT scores for a class. What is the range of these scores? 1. _____

Score	Frequency
210	1
370	5
640	7
700	4
790	1

2. Evaluate: $\sum_{k=4}^{6} (k^2 - 8)$ 2. _____

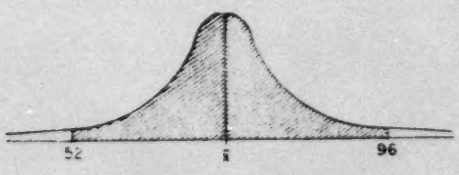
3. On a standardized test, the mean is 61 and the standard deviation is 3.2. Which score can be expected to occur less than 3% of the time? 3. _____
(1) 55 (2) 69
(3) 64 (40 5

4. What approximate percentage of the scores of a normal distribution would be expected to fall within two standard deviations from the mean? 4. _____

5. The scores on the standardized exam have a mean of 78 and a standard deviation of 3.5. Assuming a normal distribution, a student's score of 86 would rank 5. _____
(1) below the 75th percentile
(2) between the 75th and 85th percentile
(3) between the 85th and 95th percentile
(4) above the 95th percentile

6. Which statistical term represents the score which occurs most often in a distribution? 6. _____
(1) mean (3) mode
(2) median (4) standard deviation

7. In the accompanying diagram, the shaded area represents approximately 95% of the scores on a standardized test. If these scores ranged from 52 to 96, which could be the standard deviation? 7. _____
(1) 11 (3) 76
(2) 22 (4) 44



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8. A standardized test has a mean of 55 and a standard deviation of 7.2. A student in the 60th percentile would have a score of: 8. _____

9. What is the median of the following set of data? 9. _____

x_i	i
70	4
85	3
90	2
95	1

10. Evaluate: $\sum_{n=1}^{3} \sin \frac{n}{2}$ 10. _____

11. On a standardized test, the mean was 75 and the standard deviation was 4.0. Approximately what percentage of the scores would fall within the range 71 to 79? 11. _____

12. What is the mean of the following data? 12. _____

measure (x_i)	frequency (f_i)
15	3
20	2
25	5

13. For a standardized test, the mean is 30 and the standard deviation is 3.1. Which score could be expected to occur the greatest number of times? 13. _____
(1) 24.1 (3) 33.0
(2) 26.7 (4) 36.2

14. On a standardized exam, a student scores 77. The mean for the exam is 70 and the standard deviation is 3. The student's score ranks 14. _____
(1) below the 75th percentile
(2) between the 75th and the 85th percentile
(3) between the 85th and the 95th percentile
(4) above the 95th percentile

15. In a standardized test, approximately what percent of the scores will fall between +1 standard deviation and -1 standard deviation from the mean? 15. _____

16. For a standardized test, if the mean is 20 and the standard deviation is 2.6, which wcore will occur less than 5 times out of 100? 16. _____
(1) 25.1 (2) 22.6 (3) 15.7 (4) 14.6

17. On a standardized exam, the mean is 29 and the standard deviation is 2. If a student scored 34 on the exam, the student ranks 17. _____
(1) below the 7-th percentile
(2) between the 70th percentile and the 80th percentile
(3) between the 80th percentile and the 90th percentile
(4) above the 90th percentile

18. A set of measure that follows a bell curve has a mean of 50 and standard deviation of 5. Approximately what percent of the measures fall between 40 and 60? 18. _____

19. Evaluate: $2\left(\sum_{k=2}^{4} k^2\right)$ 19. _____

20. Which correctly compare the mean and median of the set of data shown in the accompanying table? 20. _____

x_i measure	f_i frequency
60	2
75	4
80	1
90	3

(1) The mean and median are euqal.
(2) The mean exceed the median by 2.
(3) The median exceeds the mean by 2.
(4) The mean exceeds the median by 2.5.

Show All Work:

21. A class of students obtained the following results on a test:
4 students received 90%
5 students received 80%
8 students received 70%
3 students received 60%
For these scores, find the:
a. mean [3]
b. standard deviation to he <u>nearest teth</u> [7]

22. On an IQ test, the scores of 5 students were 134, 131, 126, 125, and 119. Calculate the standard deviation of these scores to the <u>nearest tenth</u>. [10]

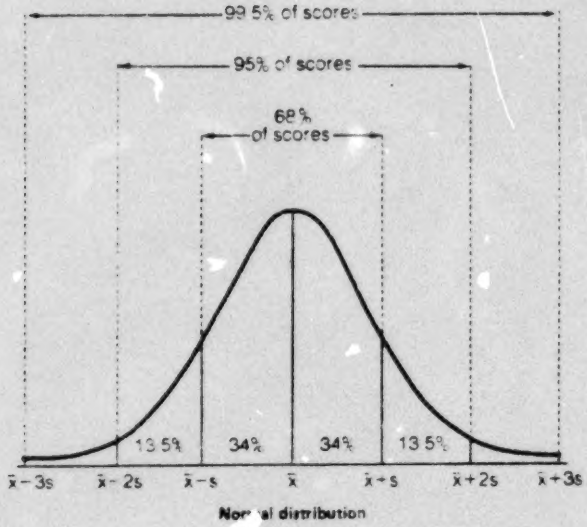
STANDARD DEVIATION AND THE NORMAL CURVE

The distribution of data represented by the normal curve is called a normal distribution. In a normal distribution, the pattern of scores below will occur, correct to the nearest one-half of one percent.

1. 68% of the scores lie within one standard deviation of the mean. That is, 68% slightly more than two-thirds of the scores, lie in the interval from $\bar{x} - s$ to $\bar{x} + s$, or one standard deviation below the mean to one standard deviation above the mean. Because the curve is symmetrical, 34% of the scores lie in the interval from $\bar{x} - s$ to x, and 34% lie in the interval from $\bar{x}$ to $\bar{x} + s$.

2. 95% of the scores lie within two standard deviation of the mean. Thus, 47.5% of the scores from $\bar{x} - 2s$ to $\bar{x}$, and 47.5% lie in the interval from $\bar{x}$ to $\bar{x} + 2s$. By arithmetic, we can show that each interval, $\bar{x} - 2s$ to $\bar{x} - s$ and $x + s$ to $\bar{x} + 2s$, contains 13.5% of the scores.

3. 99.5% of the scores lie within three standrad deviations of the mean.



Normal distribution

ANSWERS

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1: Review of Factoring

1. $18-6y-12x = 6(3-y-2x)$

2. $x^{b+1}-x^{b} = x^{b}(x-1)$

3. $(10-x)(10+x)$

4. $(y-6)^2$

5. $(3y+1)(y+1)$

6. $(x-9)(x-3)$

7. $(2x-1)(x+4)$

8. $80-5d^2 = 5(16-d^2) = 5(4-d)(4+d)$

9. $x(x^2-121) = x(x-11)(x+11)$

10. $(y^2-1)(y^2+1) = (y-1)(y+1)(y^2+1)$

11. $ax(x^2-9) = ax(x-3)(x+3)$

12. $(y^{a}-1)(y^{a}+1)$

13. $(x^2-1)(x^2-4) = (x-1)(x+1)(x-2)(x+2)$

14. $4(x^2+10x+16) = 4(x+8)(x+2)$

15. $2a(x^2-4x-6) =$

16. $3a(3y^2-7ay+2a^2) = 3a(3y-a)(y-2a)$

17. $x^{c+2}-14x^{c+1}+24x^{c} = x^{c}(x^2-14x+24) = x^{c}(x-12)(x-2)$

18. $x(x^2+3x-10) = x(x+5)(x-2)$

19. $2y(y^2+25)$

20. $x^{2k}-16 = (x^{k}-4)(x^{k}+4)$

21. $(y^2-9)(y^2+2)$ $(y-3)(y+3)(y^2+2)$

22. $5(x^2-ax-12a^2) = 5(x-4a)(x+3a)$

23. $4y(2y^2-15y-8)$ $4y(2y+1)(y-8)$

24. $bx(3x^2+bx-2b^2) = bx(3x-2b)(x+b)$

25. $(5x+2d^2)$

PAGE 2: Review of Factoring

1. $3x(x-4)$

2. $3y^2(3y+1)$

3. $(k-7)(k+7)$

4. $(x+5)^2$

5. $(y+9)(y+1)$

6. $(x+6)(X-4)$

7. $(2K-3)(K-2)$

8. $3(y^2-4) = 3(y-2)(y+2)$

9. $7(a-b)(a+b)$

10. $4c(c^2-9x^2) = 4c(c-3x)(c+3x)$

11. $4ab(b-3a)$

12. $y^{a}(y^2+2)$

13. $(ab-12)(ab+12)$

14. $(3x+1)(3x+1)$

15. $(K+7)(K-2)$

16. $(2x+1)(x+6)$

17. $(3y+1)(2y-5)$

18. $(10-2x)(10+2x)$

19. $3y(y^2-64) = 3y(y-8)(y+8)$

20. $(x^2-25)(x^2+25) = (x-5)(x+5)(x^2+25)$

21. $4(x^2-5x+6) = 4(x-3)(x-2)$

22. $(6c-10d)(6c+10d)$

23. $x^{k}(X^2-4) = x^{k}(X-2)(X+2)$

24. $2x(x^2+3x+1)$

25. $(3x+2)(x+6)$

PAGE 3: Simplifying Polynomials

1. $\frac{3}{4}x^2$

2. $\frac{2a}{y^4}$

3. $\frac{1}{3b}$

4. $\frac{3}{4}$

5. -1

6. $\frac{2(y+2)}{3(y+2)} = \frac{2}{3}$

7. $\frac{2z}{2z(z+3)} = \frac{1}{z+3}$

8. $\frac{16^{8}x^{2}}{2x(x-4)} = \frac{8x}{x-4}$

9. $\frac{bx^2(x-b)}{bx^2(x-2b)} = \frac{x-b}{x-2b}$

10. $K-5$

11. $\frac{y+9}{y-9}$

12. $\frac{(x-1)^2}{2(1-x)} = -\frac{(x-1)}{2}$

13. $\frac{3(a+1)}{(a+1)(a-1)} = \frac{3}{a-1}$

3: Simplifying Polynomials (Continued)

14. $\frac{2(p-4)}{-1(p-4)(4-p)} = \frac{-2}{4-p}$

15. $\frac{x-a}{x+a}$

16. $\frac{-x^2y(x-y)}{xy^2(y-x)} = \frac{x}{y}$

17. $\frac{(1-3x)(1+3x)}{x(3x-1)} = \frac{-(1+3x)}{x}$

18. $\frac{5(a-3)(a+3)}{5(a-3)} = a+3$

19. $\frac{(x-9)(x+9)}{(9-x)(9+x)} = -1$

20. $\frac{y(y-2)}{(y-2)(y+1)} = \frac{y}{y+1}$

21. $\frac{2y(y^2-4)}{2y(2y-4)} = \frac{2y(y-2)(y+2)}{4y(y-2)} = \frac{y+2}{2}$

22. $\frac{(x+2)(x-1)}{(x-1)(x+4)} = \frac{x+2}{x+4}$

23. $\frac{3y(y-4)}{y(y-4)(y+4)} = \frac{3}{y+4}$

24. $\frac{-1(x^2+6x-16)}{x^2-64} = \frac{-1(x-2)(x+8)}{(x-8)(x+8)} = \frac{2-x}{x-8}$

PAGE 4: Addition and Subtraction of Polynomials

1. $\frac{6}{y}$

2. $\frac{5x}{8}$

3. $\frac{7b}{3y}$

4. $\frac{1}{4a}$

5. $-\frac{1}{x}$

6. $\frac{x^2+5x-8}{x^2}$

7. $\frac{bc-a^2}{ab}$

8. $\frac{x^2+y^2}{x^2y^2}$

9. $\frac{5t+9r}{rst}$

10. 2

11. $\frac{x-4}{x-2}$

12. $\frac{z+5}{z}$

13. $\frac{2}{y(y+3)}$

14. $\frac{1}{(x+3)(x+)} + \frac{1}{(x+1)(x-1)} = \frac{x-1 + x+3}{(x+3)(x-1)(x+1)} = \frac{2(x+1)}{(x+3)(x-1)(x+1)} = \frac{2}{x^2+2x-3}$

15. $\frac{7}{(y-7)(y+7)} - \frac{6}{(y-7)(y+5)} = \frac{7(y+5)(-6(y+7)}{(y-7)(x+7)(y+5)} = \frac{1}{(y+7)(y+5)}$

16. $\frac{10x}{(x-5)(x+2)} - \frac{5(x+5)}{(x-5)(x-5)} = \frac{5}{x+5}$

17. $\frac{9}{(x+5)(x+2)} + \frac{3(x+2)}{(x-5)(x+5)} - \frac{1(x+5)}{(x+5)(x+2)} = \frac{2}{x+2}$

18. $\frac{2(2y+1)-1(2y-1)-2}{(2y+1)(2y-1)} = \frac{1}{2y-1}$

19. $\frac{y^2-1}{y}$

20. $\frac{2x(x-a)+ax}{x-a} = \frac{2x^2-2ax+ax}{x-a} = \frac{x(2x-a)}{x-a}$

21. $\frac{(y+3)(y-2)-(y-6)}{y-2} = \frac{y^2}{y-2}$

22. $\frac{y^2}{y+1}$

23. $\frac{2x-1+(x+2)(2x-3)}{x+2} = \frac{2x^2+3x-7}{x+2}$

24. $\frac{8x}{4x-1} - \frac{2x}{x(4x-1)} = \frac{2(4x-1)}{4x-1} = 2$

25. $\frac{y}{5-y} - \frac{5y}{y(5-y)} = -1$

PAGE 5: Multiplication/Division of Polynomials

1. $\frac{x}{2}$

2. $\frac{1}{k}$

3. $\frac{2}{3a}$

4. $\frac{2a}{5b^2}$

5. 1

6. K

7. $\frac{1}{3}$

8. $\frac{(x+4)}{4}$

9. $\frac{x+4}{4x}$

10. y/c

11. $\frac{x-5}{x}$

5: Multiplication/Division of Polynomials (Continued)

12. $\frac{3}{5b}$

13. $\frac{x^2+1}{x+1}$

14. $\frac{y(y+1)}{y-1}$

15. $\frac{3a-1}{3a}$

16. -1

17. $\frac{1}{2}$

18. -1

19. $\frac{2}{x-1}$

1

PAGE 6: Simplifying Complex Fractions

1. ab

2. $\frac{2}{K+2}$

3. $\frac{3}{2}$

4. $\frac{2}{3}$

5. $\frac{1}{1-d}$

6. $\frac{2y-1}{2y+1}$

7. $\frac{x}{x+2}$

$\frac{5}{5z-1}$

9. $\frac{b+7}{7}$

10. $\frac{m+r}{m-r}$

11. -1

12. a+b

13. $\frac{5}{x}$

14. b

15. $\frac{y^2+9}{y+3}$

16. y

17. b+2

18. $\frac{x^2+y^2}{2xy}$

19. $\frac{4}{x}$

20. $\frac{2}{5}$

21. a) $\frac{1}{x+7}$

b) -y

22. $\frac{1}{3x-1}$

23. $\frac{a+b}{b}$

24. $\frac{2}{t-2}$

25. 2(x+4)

26. $\frac{1}{2a^2}$

27. $\frac{-1}{n}$

28. $\frac{(x+1)}{(x-3)}$

29. $\frac{y-4}{y-2}$

30. $\frac{K-1}{K-2}$

PAGE 7: Solving Fractional Equations

1. x=2

2. x=3

3. x=2

4. x=3

5. y=3

6. a=4

7. x=1

8. x=3

9. $a=-\frac{1}{3}$

10. y=3

11. b=1

12. x=2

13. x=3

14. t=-1

15. x=4

16. $r=\frac{1}{5}$

17. m=2

18. $x=\frac{1}{2}$

19. d=2

20. $y=-\frac{1}{2}$

PAGE 8: Solving Fractional Equations

1. y=5

2. x=30

3. y=3

4. a=2

5. $x=\frac{1}{4}$

6. x=5

8: Solving Fractional Equations (Continued)

7. $t=3$
8. $x=4$
9. $z=1$
10. $a=-10$
11. $x=\frac{1}{2}$
12. $x=-7$
13. $c=\frac{1}{2}$
14. $y=5$
15. $b=-4$
16. $y=-8$
17. $x=\frac{1}{2}$
18. $c=6$
19. $x=3$
20. $x=-6$

REGENTS REAL NUMBERS

Part 1

1. (2)
2. 3
3. b
4. (3)
5. (4)
6. $x=11$
7. (4)
8. $a+2$
9. (4)
10. (4)
11. (2)
12. $\frac{3x}{x+3}$
13. b
14. $x=0, x=4$
15. (2)
16. (3)
17. $x=6$
18. $\frac{x+2}{x}$
19. $x-2$
20. $\frac{1-x}{x}$

Part 2

1. $\{2,-1\}$
2. $x=\frac{1}{2}$
3. -1
4. $x=3$
5. (1)
6. $\frac{3}{x}$
7. $-6x$
8. (4)
9. (2)
10. 2
11. (4)
12. (D)
13. $\frac{b}{a}$
14. (3)
15. $\frac{1}{x+1}$
16. $\frac{x+2}{4}$
17. (1)
18. (3)
19. (4)

PLIFYING RADICALS
E 15

1. $8\sqrt{2}$
2. $-10\sqrt{5}$
3. $10\sqrt{6}$
4. $-42\sqrt{2}$
5. $\frac{3\sqrt{2}}{2}$
6. $-2\sqrt{5}$
7. $3n\sqrt{m}$
8. $-4ab\sqrt{2b}$
9. $10xy\sqrt{xy}$
10. $-14ab\sqrt{6ab}$
11. $\frac{2\sqrt{15}}{5}$
12. $\frac{-\sqrt{21}}{6}$
13. $\frac{2\sqrt{14}}{7}$
14. $\frac{-\sqrt{10}}{6}$
15. $10\sqrt{2}$
16. $-\frac{\sqrt{6}}{8}$
17. $2\sqrt{6c}$
18. $-20\sqrt{3}$
19. $5\sqrt{5}$
20. $-4x\sqrt{2}$

ADDITION & SUBTRACTION OF
DICALS PAGE 16

1. $4\sqrt{3}$
2. $3\sqrt{2} + 4\sqrt{5}$
3. $8\sqrt{y}$
4. $6\sqrt{2}$
5. $8\sqrt{5}$
6. $\sqrt{2}$
7. $11\sqrt{2}$
8. $19\sqrt{3}$
9. $5\sqrt{2}$
10. 0
11. $-2\sqrt{7}$
12. $12\sqrt{x}$
13. $6\sqrt{x}$
14. $8\sqrt{3}$
15. $3\sqrt{5}$
16. $-\sqrt{3}$
17. 0
18. $3\sqrt{10}$
19. $\sqrt{2} + \sqrt{3}$
20. 0

MULTIPLY/DIVIDE
RADICALS PAGE 17

1. $3\sqrt{7}$
2. $70\sqrt{6}$
3. $10\sqrt{10}$
4. $42\sqrt{7}$
5. $10\sqrt{3}$
6. $9\sqrt{6}$
7. $12\sqrt{6}$
8. $9\sqrt{2}$
9. $36\sqrt{2}$
10. 48
11. $2\sqrt{3}$
12. $3\sqrt{2}$
13. $6\sqrt{2}$
14. $5\sqrt{2}$
15. $\frac{5\sqrt{5}}{2}$
16. $\frac{\sqrt{30}}{5}$
17. $3\sqrt{2}$
18. $3\sqrt{15}$
19. $\frac{5-\sqrt{35}}{5}$ or $1-\frac{\sqrt{35}}{5}$
20. $\frac{12+2\sqrt{3}}{3}$ or $4+\frac{2\sqrt{3}}{3}$

SOLVING RADICAL EQUATIONS
PAGE 18 [Step-by-Step Solutions
and Checks on following pages]

1. x=32
2. No Roots
3. r=22
4. b=52
5. $r = \frac{1}{4}$
6. $x = 20$
7. b=65
8. a=11
9. m=64
10. x=15
11. x=25
12. x=25
13. x=4
14. x=4
15. y=2
16. x=12

VING RADICAL EQUATIONS
E 18

1. $\sqrt{2x} = 8$

$2x = 64$

$x = 32$

$\sqrt{2x} = 8$

$\sqrt{64} = 8$

$8 = 8$

2. $4\sqrt{3x} = -9$

$48x = 81$

$x = \frac{81}{48}$

$x = \frac{27}{16}$

No Root

$4\sqrt{3x} = -9$

$4\sqrt{3\frac{27}{16}} = -9$

$4\sqrt{\frac{81}{16}} = -9$

$\sqrt{81}x = -9$

3. $\sqrt{r+3} = 5$

$r + 3 = 25$

$r = 22$

$\sqrt{r+3} = 5$

$\sqrt{25} = 5$

$5 = 5$ ✓

4. $\sqrt{2b-4} = 10$

$2b - 4 = 100$

$2b = 104$

$b = 52$

$\sqrt{2b-4} = 10$

$\sqrt{104-4} = 10$

$\sqrt{100} = 10$

$10 = 10$ ✓

5. $\sqrt{2r} + 7 = 8$

$\sqrt{2r} = 1$

$2r = 1$

$r = \frac{1}{2}$

$\sqrt{2r} + 7 = 8$

$\sqrt{1} + 7 = 8$

$1 + 7 = 8$

$8 = 8$ ✓

6. $2\sqrt{5x} = 20$

$\sqrt{5x} = 10$

$5x = 100$

$x = 20$

$2\sqrt{5x} = 20$

$2\sqrt{100} = 20$

$2\ 10 = 20$

$20 = 20$ ✓

7. $\sqrt{b-1} = 8$

$b - 1 = 64$

$b = 65$

$\sqrt{b-1} = 8$

$\sqrt{64} = 8$

$8 = 8$

8. $\sqrt{3a+3} = 6$

$3a + 3 = 36$

$3a = 33$

$a = 11$

$\sqrt{3a+3} = 6$

$\sqrt{33+3} = 6$

$36 = 6$

$6 = 6$ ✓

9. $\sqrt{m-6} = 2$

$\sqrt{m} = 8$

$m = 64$

$\sqrt{m-6} = 2$

$\sqrt{64-6} = 2$

$8 - 6 = 2$

$2 = 2$ ✓

10. $\sqrt{x+1} = 4$

$x + 1 = 16$

$x = 15$

$\sqrt{x+1} = 4$

$\sqrt{16} = 4$

$4 = 4$ ✓

11. $\sqrt{2x-1} = 7$

$2x - 1 = 49$

$2x = 50$

$x = 25$

$\sqrt{2x-1} = 7$

$\sqrt{50-1} = 7$

$\sqrt{49} = 7$

$7 = 7$ ✓

12. $\sqrt{x} + 3 = 8$

$\sqrt{x} = 5$

$x = 25$

$\sqrt{x} + 3 = 8$

$\sqrt{25} + 3 = 8$

$5 + 3 = 8$

$8 = 8$ ✓

13. $2\sqrt{x} + 1 = 5$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

$2\sqrt{x} + 1 = 5$

$2\sqrt{4} + 1 = 5$

$4 + 1 = 5$

$5 = 5$ ✓

14. $\sqrt{3x-8} = \sqrt{x}$

$3x - 8 = x$

$2x = 8$

$x = 4$

$\sqrt{3x-8} = \sqrt{x}$

$\sqrt{12-8} = \sqrt{4}$

$\sqrt{4} = 2$

$2 = 2$ ✓

15. $\sqrt{5y} = \sqrt{2y+6}$

$5y = 2y + 6$

$3y = 6$

$y = 2$

$\sqrt{5y} = \sqrt{2y+6}$

$\sqrt{10} = \sqrt{y+6}$

$\sqrt{10} = \sqrt{10}$ ✓

16. $\sqrt{2x+1} - 1 = 4$

$\sqrt{2x+1} = 5$

$2x + 1 = 25$

$2x = 24$

$x = 12$

$\sqrt{2x+1} - 1 = 4$

$\sqrt{24+1} - 1 = 4$

$\sqrt{25} - 1 = 4$

$5 - 1 = 4$

$4 = 4$ ✓

ITION/SUBTRACTION OF COMPLEX NUMBERS
19

1. $15 + 11i$
2. $10 - 8i$
3. $1 + 0i$
4. $4 + 2i$
5. $-13 + 12i$
6. $0 + 3i$
7. $4.2 + 2.3i$
8. $1.7 - .2i$
9. $\frac{5}{6} - \frac{3i}{4}$
10. $\frac{1}{2} - \frac{3i}{8}$
11. $-2 + i$
12. $6 + 5i\sqrt{3}$
13. $-4 - 6i\sqrt{5}$
14. $0 - 15i\sqrt{2}$
15. 0
16. $5 + 8i$
17. $-3 + 2i$
18. $14 - i$
19. $7 - 14i\sqrt{2}$

MULTIPLICATION OF COMPLEX NUMBERS
PAGE 20

1. $11 + 7i$
2. $16 + 2i$
3. $5 + 5i$
4. $13 - 6i$
5. $5 - 15i$
6. $19 - 23i$
7. $-4 - 18i$
8. $37 + 0i$
9. $10 + 0i$
10. $11 + 18i$
11. $34 - 2i$
12. $12 + 5i$
13. $-50 - 27i$
14. $5 - 12i$
15. $-16 - 16i$
16. $0 + 37i$
17. $0 - 25i$
18. $16 - 10i$
19. $-1 + 0i$
20. $9 + 8i$

DIVISION OF COMPLEX NUMBERS
PAGE 21

1. $3 - i$
2. $6 + i$
3. $1 - 5i$
4. $4 + 3i$
5. $9 + 2i$
6. $7 - 5i$
7. $4 + 4i$
8. $5 - 5i$
9. $\frac{23 + i}{53}$
10. $-5 - 3i$
11. $\frac{12 + 5i}{13}$
12. $\frac{1}{2} - \frac{7i}{4}$
14. $3 + 4i$
15. $\frac{5 + 14i}{13}$
16. $\frac{24 - 7i}{25}$
17. $\frac{35 + 12i}{37}$
18. $\frac{7 + i}{10}$
19. $4 + i$
20. $3 + 2i$

ROOTS OF QUADRATIC EQUATIONS
PAGE 22

1. (1)
2. (4)
3. (4)
4. (3)
5. (2)
6. (4)
7. (2)
8. (3)
9. $x^2 - 64 = 0$
10. $x^2 - 7x = 0$
11. $x^2 - 3 = 0$
12. $2x^2 - 9x + 4 = 0$
13. $2x^2 + x - 6 = 0$
14. $9x^2 + 9x + 2 = 0$
15. $x^2 + 1 = 0$
16. $x^2 + 16 = 0$
17. $x^2 - 4x + 5 = 0$
18. $x^2 - 12x + 27 = 0$
19. $x^2 - 4x + 1 = 0$
20. $x^2 - 2x - 6 = 0$

1. 2 roots; irrational and unequal (Discriminant = 29)
2. 2 roots; rational and unequal (Discriminant = 49)
3. 2 roots; imaginary (Discriminant = -7)
4. 2 roots; rational and equal (Discriminant = 0)
5. 2 roots; imaginary (Discriminant = -4)
6. 2 roots; rational and unequal (Discriminant = 169)
7. 2 roots; rational and unequal (Discriminant = 25)
8. 2 roots; irrational and unequal (Discriminant = 41)
9. 2 roots; rational and equal (Discriminant = 0)
10. 2 roots; irrational and unequal (Discriminant = 37)

11. $x^2 - 6 + 13 = 0$

$$x = \frac{6 \pm \sqrt{36 - 52}}{2}$$

$$x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$$

$$x = 3 + 2i,\ 3 - 2i$$

12. $x^2 = 2x + 6$

$$x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 24}}{2}$$

$$x = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2}$$

$$x = 1 + \sqrt{7},\ 1 - \sqrt{7}$$

13. $9x^2 + 4 = 0$

$$x = \frac{0 \pm \sqrt{0 - 144}}{18}$$

$$x = \frac{\pm 12i}{18} = \frac{\pm 2i}{3}$$

$$x = \frac{2i}{3},\ -\frac{2i}{3}$$

14. $x^2 + 4x = x$ or $x 2 + 4x = x$

$x^2 + 3x = 0$ $x^2 + 3x = 0$

$$x = \frac{-3 \pm \sqrt{9 - 0}}{2}$$ $x (x + 3) = 0$

$\{-3, 0\}$

$$x = \frac{-3 \pm 3}{2}$$

$$x = 0, -3$$

15. $x^2 + 20x = 6x - 49$

$x^2 + 14x + 49 = 0$

$(x + 7)(x + 7) = 0$

$x = -7$

ING QUADRATIC EQUATIONS

1. $x^2 - 6x + 10 = 0$

$x = \frac{6^+ - \sqrt{36 - 40}}{2}$

$x = \frac{6^+ - \sqrt{-4}}{2} = \frac{6^+ - 2i}{2}$

$x = 3 + i, 3 - i$

2. $2x^2 = 6x - 5$

$2x^2 - 6x + 5 = 0$

$x = \frac{6^+ - \sqrt{36 - 40}}{4}$

$x = \frac{6^+ - \sqrt{-4}}{4} = \frac{6^+ - 2i}{4}$

$x = \frac{3 + i}{2} \quad \frac{3 - i}{2}$

3. $x^2 + 10x + 26 = 0$

$x = \frac{-10^+ - \sqrt{100 - 104}}{2}$

$x = \frac{-10^+ - \sqrt{-4}}{2} = \frac{-10^+ - 2i}{2}$

$x = -5 + i, -5 - i$

4. $\frac{16x}{7} = \frac{7}{x} = 0$

$16x^2 + 49 = 0$

$x = \frac{0^+ - \sqrt{0 - 3136}}{32}$

$x = \frac{^+ - \sqrt{-3136}}{32} = \frac{^+ -56}{32}$

$x = \frac{7i}{4} \quad \frac{-7i}{4}$

5. $\frac{x^2}{4} + 29 = 4x$

$x^2 - 4x + 29 = 0$

$x = \frac{4^+ - \sqrt{16 - 11\ 6}}{2}$

$x = \frac{4^+ - \sqrt{-100}}{2} = \frac{4^+ - 10i}{2}$

$x = 2 + 5i, 2 - 5i$

6. $x^2 = 3x - 10$

$x^2 - 12x + 40 = 0$

$x = \frac{12^+ - \sqrt{144 - 160}}{2}$

$x = \frac{12^+ - \sqrt{-16}}{2} = \frac{12^+ - 4i}{2}$

$x = 6 + 2i, 6 - 2i$

KS FOR SOLVING QUADRATIC EQUATIONS

1. $x^2 - 6x + 10 = 0$

$(3 + i)^2 - 6(3+i) + 10 = 0$

$8 + 6i - 18 - 6i + 10 = 0$

$- 10 + 10 = 0$

$0 = 0$

$(3 - i)^2 - 6(3 - i) + 10 = 0$

$8 - 6i - 18 + 6i + 10 = 0$

$- 10 + 10 = 0$

$0 = 0$ ✓

2. $2x^2 = 6x - 5$

$2\left(\frac{3 + i}{2}\right)^2 = 6\left(\frac{3 + i}{2}\right) - 5$

$2\left(\frac{8 + 6i}{4}\right) = 9 + 3i - 5$

$4 + 3i = 4 + 3i$

$2\left(\frac{3 - i}{2}\right)^2 = 6\left(\frac{3 - i}{2}\right) - 5$

$2\left(\frac{8 - 6i}{4}\right) = 9 - 3i - 5$

$4 - 3i = 4 - 3i$ ✓

3. $x^2 + 10x + 26 = 0$

$(-5 + i)^2 + 10(-5 + i) + 26 = 0$

$24 - 10i - 50 + 10i + 26 = 0$

$- 26 + 26 = 0$

$0 = 0$

$(-5 - i)^2 + 10(-5 - i) + 26 = 0$

$24 + 10i - 50 - 10i + 26 = 0$

$- 26 + 26 = 0$

$0 = 0$ ✓

4. $\frac{16x}{7} + \frac{7}{x} = 0 \quad \frac{16x}{7} = \frac{7}{x} = 0$

$\frac{16(7i)}{7} + \frac{7}{7i} = 0 \quad 16\left(\frac{-7i}{4} + \frac{7}{-7i}\right)$

$4i + \frac{4}{i} = 0 \qquad -4i + \frac{-4}{i}$

$4i + \frac{4i}{-1} = 0 \qquad -4i + \frac{-4i}{-1} = 0$

$0 = 0$ ✓ $\qquad 0 = 0$ ✓

5. $x^2 + 29 = 4x$

$(2 + 5i)^2 + 29 = 4(2 + 5i)$

$-21 + 20i + 29 = 8 + 20i$

$8 + 20i = 8 + 20i$

$(2 - 5i)^2 + 29 = 4(2 - 5i)$

$-21 - 20i + 29 = 8 - 20i$

$8 - 20i = 8 - 20i$ ✓

6. $\frac{x^2}{4} = 3x - 10$

$\frac{(6 + 2i)^2}{4} = 3(6 + 2i) - 10$

$\frac{32 + 24i}{4} = 18 + 6i - 10$

$8 + 6i = 8 + 6i$

$\frac{(6 - 2i)^2}{4} = (6 - 2i) - 10$

$\frac{32 - 24i}{4} = 18 - 6i - 10$

$8 - 6i = 8 - 6i$ ✓

1. (1)
2. (3)
3. (1)
4. (3)
5. (1)
6. (1)
7. (3)
8. $\frac{4 + 3i}{5}$
9. $5 + i$
10. $18 + i$
11. $5 - 12i$
12. $c = 5$
13. $19i$
14. $13 - 6i$
15. $(x - 3 - i)\ (x - 3 + i) = 0$

$x^2 - -3x + xi - 3x + 9 - 3i - xi + 3i + 1 = 0$

$x^2 - 6x + 10 = 0$

16. $x^2 - 2x + 5 = 0$

$x = \frac{2^{+} - \sqrt{4 - 20}}{2}$

$x = \frac{2^{+} - \sqrt{-16}}{2} = \frac{2^{+} - 4i}{2}$

$x = 1 + 2i,\ 1 - 2i$

$x^2 - 2x + 5 = 0$

$(1 + 2i)^2 - 2(1 + 2i) + 5 = 0$

$-3 + 4i - 2 - 4i + 5 = 0$

$-5 + 5 = 0$

$0 = 0$

$(1 - 2i)^2 - 2(1 - 2i) + 5 = 0$

$-3 - 4i - 2 + 4i + 5 = 0$

$-5 + 5 = 0$

$0 = 0$

17. $x^2 = x - 1$

$x^2 - 2x + 2 = 0$

$x = \frac{2^{+} - \sqrt{4 - 8}}{2}$

$x = \frac{2^{+} - \sqrt{-4}}{2} = \frac{2^{+} - 2i}{2}$

$x = 1 + i,\ 1 - i$

18. $2x^2 - 6x + 5 = 0$

$x = \frac{6^{+} - \sqrt{36 - 40}}{4}$

$x = \frac{6^{+} - \sqrt{-4}}{4} = \frac{6^{+} - 2i}{4}$

$x = \frac{3 + i}{2},\ \frac{3 - i}{2}$

REGENTS REVIEW - COMPLEX NUMBERS PAGES 27 - 28

1. $\frac{15 + 5\sqrt{3}}{6}$
2. $3 - i$
3. $19i$
4. $3\sqrt{3}$
5. $18 + i$
6. $10 - 5i$
7. $c = 2$
8. $-8i$
9. $4i\sqrt{2}$
10. 8
11. (1)
12. (4)
13. (3)
14. (4)

15. $x^2 + 8x + 20 = 0$

$x = \frac{-8 \pm \sqrt{64 - 80}}{2}$

$x = \frac{-8 \pm \sqrt{-16}}{2} = \frac{-8 \pm 4i}{2}$

$x = -4 + 2i, -4 - 2i$

$x^2 + 8x + 20 = 0$

$(-4 + 2i)^2 + 8(-4 + 2i) + 20 = 0$

$12 - 16i - 32 - 16i + 20 = 0$

$-20 + 20 = 0$

$0 = 0$

$(-4 - 2i)^2 + 8(-4 - 2i) + 20 = 0$

$12 + 16i - 32 - 16i + 20 = 0$

$-20 + 20 = 0$

$0 = 0$

16. $(x - 2i)(x + 2i) = 0$

$x^2 - 4i^2 = 0$

$x + 4 = 0$

17. $3x^2 + 3 = 5x$

$3x^2 - 5x + 3 = 0$

$x = \frac{5 \pm \sqrt{25 - 36}}{6}$

$x = \frac{5 \pm \sqrt{-11}}{6}$

$x = \frac{5 + i\sqrt{11}}{6} \quad \frac{5 - i\sqrt{11}}{6}$

$3x^2 + 3 = 5x$

$3\left(\frac{5 + i\sqrt{11}}{6}\right)^2 + 3 = 5\left(\frac{5 + i\sqrt{11}}{6}\right)$

$3\left(\frac{14 + 10i\sqrt{11}}{36}\right) + 3 = \frac{25 + 5i\sqrt{11}}{6}$

$\frac{7 + 5i\sqrt{11}}{6} + \frac{18}{6} = \frac{25 + 5i\sqrt{11}}{6}$

$\frac{25 + 5i\sqrt{11}}{6} = \frac{25 + 5i\sqrt{11}}{6}$ ✓

18. $x^2 - 2x + 10 = 0$

$x = \frac{2 \pm \sqrt{4 - 40}}{2}$

$x = \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm 6i}{2}$

$x = 1 + 3i, 1 - 3i$

$x^2 - 2x + 10 = 0$

$(1 + 3i)^2 - 2(1 + 3i) + 10 = 0$

$-8 + 6i - 2 - 6i + 10 = 0$

$-10 + 10 = 0$

$0 = 0$ ✓

$(1 - 3i)^2 - 2(1 - 3i) + 10 = 0$

$-8 - 6i - 2 + 6i + 10 = 0$

$-10 + 10 = 0$

$0 = 0$

1.	a. constant	b. 2	c. 2
2.	a. linear	b. 3	c. 1/2
3.	a. step	b. 3	c. 0
4.	a. quadratic	b. 18	c. 1/2
5.	a. linear	b. -1	c. 3/2
6.	a. absolute value	b. 1	c. 3/2
7.	a. step	b. -1	c. 1
8.	a. quadratic	b. 6	c. -1/4
9.	a. constant	b. π	c. π

10. $-8 \leq y \leq 4$

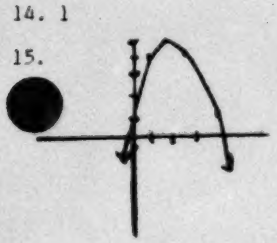
11. $-3 \leq f(x) \leq 5$

12. $g \leq 6$

13. 17

14. 1

15.



x	f (x)
0	1
1	4
2	5
3	4
4	1

16. $x > 2 \wedge x < 2$

17. $x > 9 \wedge x < 9$

18. $x > -7 \wedge x < -7$

19. a. all reals
    b. all reals

20. a. all reals
    b. all positive reals and 0 ($y \geq 0$)

21. a. all non-negative reals
    b. all reals

22. a. all reals
    b. all non-negative reals $h(x) \geq 0$

23. a. all reals
    b. $f(x) \leq 10$

24. a. all reals
    b. all reals

APPLICATIONS WITH FUNCTIONS PAGE 30

1. a. \$1.40 b. \$1.05 c. \$2.24 d. \$3.99 e. 38

2. a. \$.04 b. \$.07 c. \$.03 d. none e. .80
   f. TRUE g. FALSE f (\$.10) = NONE while .08 (.10) = .0080 (rounded) (.01)

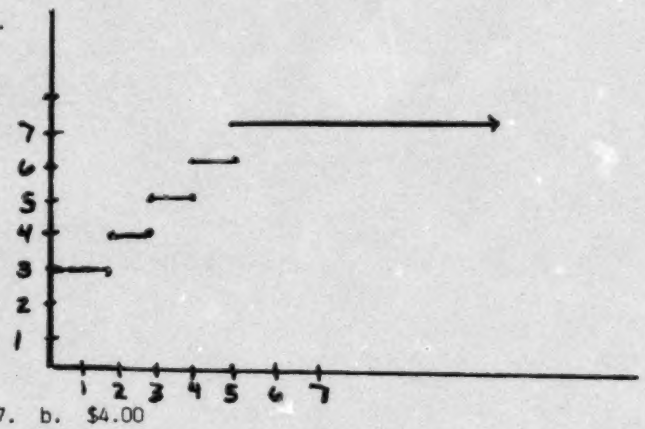
3. a. \$7.95 b. \$8.95 c. \$7.45 d. \$13.20 e. 27

4. a. \$3.90 b. \$5.15 c. \$7.65 d. \$6.40 e. 250
   f. 201

5. a. 16 b. 64 c. 144 d. 256 e. 784 ft.

6. a. T b. \$70 c. weekly rental is less
   d. $50 < r \leq 100$

7.



   b. \$4.00
   c. \$6.00
   d. \$7.00
   e. \$7.00

8. a. -
   b. T
   c. \$.10
   d. \$.14
   e. \$.63

INVERSE FUNCTIONS
PAGE 32

1. $\{(5,1), (7,2), (-2,3), (-3,4)\}$
2. $\{(6,0), (2,4), (\square, -1), (8, -2)\}$
3. $\{(1,1), (2,2), (3,3), (4,4)\}$
4. $\{(k,1), (k+1,2), (k+2,3)\}$
5. a. $\{(-2,3), (-2,4), (-1,5)\}$

   b. NO; -2 has 2 different 2nd coordinates

6. $y = 1/3x$
7. $y = 3x - 3$

   $= 3(x - 1)$
8. $y = x^2$
9. $y = x + 6$
10. $y = \frac{x + 8}{4}$
11. $y = \frac{x - 12}{3}$
12. $= 4x$
13. $y = 5 - x$
14. $y = 3\sqrt{x + 7}$
15. a. -1

    b. 2
16. a. 2

    b. -3
17. 123
18. SEE DIAGRAMS

    c. D: $-1 \leq x \leq 4$ R: $-2 \leq y \leq 3$

    d. $-2 \leq x \leq 3$ $-1 \leq y \leq 4$
19. c. D: $0 \leq x \leq 6$ R: $-2 \leq y \leq 2$

    D: $-2 \leq x \leq 2$ R: $0 \leq y \leq 6$
20. c. Domain, Range: all x, all y

    d. Domain, Range: all x, all y

FUNCTION COMPOSITION
PAGE 33

1. $6x - 12$ or $6(x - 2)$
2. $4x - 10$
3. $2x + 5$
4. $x - 8$
5. $10x$
6. $3x - 7$
7. $2x$
8. $3 - x$
9. $x^2 - 10 + 25$ or $(x - 5)^2$
10. $4x - x^2$
11. (2)
12. $4x + 15$
13. a. $x^2 - 4x + 3$ for $x = 4; 3$

    b. 0

    c. -1

    d. 0

    e. 15

    f. $x^2 - 4x + 3$
14. a. $2x + 5$

    b. $2x + 1$

    c. $2(x - 2)$

    d. $2x + 1$

    e. yes - composition is associative
15. a. $\frac{1}{|x|}$

    b. $[x - 3]$

    c. $|x - 3|$

    d. $[x] - 3$

    e. $\frac{1}{x} - 3$

    f. $2(x - 3)$

    g. $\frac{1}{2x - 3}$

    h. $\left[\frac{2}{x}\right]$

    i. $2|x| - 3$

    j. $\left|\frac{1}{2x}\right| = \frac{1}{|2x|}$

    $2 \cdot 1 \cdot 2$

    $2 \cdot 1 \cdot 2$
16. a. True

    b. $.08

    c. $1.01

    d. True

    e. $3.34

    f. $21.15

FUNCTIONS: DEFINITIONS, INVERSES
REGENTS ?'s PAGE 34

1. 7
2. -16
3. (4)
4. (3)
5. (1)
6. (1)
7. (2)
8. (3)
9. (3)
10. (2)
11. (1)
12. (2)
13. (4)
14. (4)
15. (3)
16. (2)

RATIONAL EXPONENTS
PAGE 36

1. a. $3^{23/20}$ b. $10^{3/2}$
2. a. $\sqrt{3} - \sqrt{2}$ b. 2
3. a. $2^{3\sqrt{3}}$ b. 3^{5-2}
4. a. 1 b. 1
5. a. 5 2/3 b. 25
6. a. $7^{\sqrt{3}}$ b. $4^5 = 2^{2\ 5}$
7. a. $x = 10$
   b. $x = \frac{\sqrt{2}}{6}$
   c. $x = \frac{3}{4}$
   d. $x = 2$
   e. $x = \frac{-6}{5}$
   f. $x = 3$
8. a. $x = \frac{-4}{3}$
   b. $x = \frac{-1}{3}$
   c. $x = \frac{3}{2}\sqrt{3} + 1$ or $x = \frac{3\sqrt{3} + 2}{2}$
   d. $x = 14$
   e. $x = \frac{2}{5}$
   f. $x = \frac{5}{2}$

GRAPHING EXPONENTIAL FUNCTIONS
PAGE 37

3. b. Positive reals
   c. I, II
4. c $y = \frac{5}{2} - x$
5. (2)
6. (1)
7. (3)
8. (3)
9. (1)

LOGARITHMIC, EXPONENTIAL NOT.
PAGE 38

1. a. $2^4 = 16$
   b. $10^4 = 10{,}000$
   c. $9^{.5} = 3$
   d. $x^3 = 28$
   e. $x = 6^{-3}$
   f. $10x = 119$
2. a. $3^5 = 243$
   b. $2^{-6} = \frac{1}{64}$
   c. $(1/2)^2 = 1/4$
   d. $x^2 = 53$
   e. $(1/4)^{-5} = x$
   f. $5^x = 33$
3. a. $\log_8 512 = 3$
   b. $\log_3 1/81 = -4$
   c. $\log_{27} 3 = 1/3$
   d. $\log_x 1/128 = -6$
   e. $\log_7 x = 4$
   f. $\log_{10} 139 = x$
4. a. $\log_6 1296 = 4$
   b. $\log_5 1/125 = -3$
   c. $\log_8 2 = 1/3$
   d. $\log_x 1/256 = -4$
   e. $\log_3 x = 8$
   f. $\log_5 168 = x$
5. $x = 4$
6. $x = 3$
7. $x = -3$
8. $x = 64$
9. $x = 1/9$
10. $x = 8$
11. $x = 2\sqrt{2}$
12. $x = 1/2$
13. $x = 8$
14. $x = 2$

LOGARITHM TABLES
PAGE 39

1. a. 6.6561
   b. 6.3222 - 10
   c. 7.5490
   d. 8.8633 - 10
   e. 3.7782 - 10
   f. 3.5051

2. a. 7.8669
   b. 5.3054
   c. 8.4786 - 10
   d. 2.0414 - 10
   e. 1.4346
   f. 8.9805

3. a. .0000586
   b. .000195
   c. 4,620,000
   d. 24.3
   e. .00945
   f. 100

4. a. .000446
   b. .0000000645
   c. 131
   d. 533,000
   e. 10
   f. .000644

5. a. 4.8940
   b. 2.1418
   c. 9.9130 - 10
   d. 1.1504
   e. 7.7895 - 10
   f. 3.4527

6. a. 7087
   b. 2.6718
   c. 8.9310 - 10
   d. 3.8031
   e. 4.4980
   f. 1.8361

7. a. 207,400
   b. .001297
   c. .02978
   d. 166.4
   e. .0003078
   f. 94.68

8. a. 41,340
   b. .01593
   c. .005456
   d. 594.6
   e. .00006496
   f. 19.91

CALCULATIONS USING LOGS
PAGE 40

1. x = 24
2. x = 6
3. x = 6
4. x = 2
5. x = 4
6. x = 3
7. x = 3
8. x = 49
9. 497300
10. 843500
11. 5041000
12. 4506000
13. 3193
14. -4052
15. - .0515
16. - .0566
17. - .0003
18. .0450
19. 250600000000
20. 38880000000
21. 20.67
22. 7.630
23. -55.00
24. 10.95
25. 16.99
26. -50.87
27. 4.930
28. .0143
29. 73210000
30. 284.9
31. 922500000000000
32. 174200000000
33. 66.92
34. 28090000
35. 1.607
36. 1.318
37. 85.67
38. .0728
39. 3.036
40. 11.06
41. 9.120
42. 1.998
43. 427.9
44. 6993
45. 32.14
46. 6.494
47. 1.635
48. 3.368
49. -6.814
50. 10.68
51. 7.409
52. 23.49
53. 1.064
54. 1.672

(PROBLEMS 1,2 & 3 in detail on next 3 pages - 4th page contains rest of answers.)

1. $\gamma = 59° 30°$

$$\frac{a}{\sin a} = \frac{b}{\sin b}$$

$$\frac{5390}{\sin 37° 40'} = \frac{b}{\sin 82° 50'}$$

b. $\sin 37° 40' = 5390 \sin 82° 50'$
$\log b + \log \sin 37° 40' = \log 5350 + \log \sin 82° 50'$
$\log b + (9.7861 - 10) = 3.7316 + (9.9966 - 10)$
$\log b + (9.7861 - 10) = 13.7282 - 10$
$\log b = 3.9421$
$b = 8750$ (if interpolate .8752)

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}$$

$$\frac{5390}{\sin 37° 40'} = \frac{c}{\sin 59° 30'}$$

$c \sin 37° 40' = 5390 \sin 9° 30'$
$\log c + \log \sin 37° 40' = \log 5390 + \log \sin 59° 30'$
$\log c + (9.7861 - 10) = 3.7316 + (9.9353 - 10)$
$\log c + (9.7861 - 10) = 13.6669 - 10$
$\log c = 3.8808$
$c = 7600$

2. $\alpha = 71° 30'$

$$\frac{c}{\sin \gamma} = \frac{b}{\sin \beta}$$

$b \sin \gamma = c \sin \beta$
$\log b + \log \sin \gamma = \log c + \log \sin \beta$
$\log b + \log \sin 65° 20' = \log 974 + \log \sin 43° 10'$
$\log b + (9.9584 - 10) = 2.9886 + (9.8351 - 10)$
$\log b + (9.9584 - 10) = 12.8237 - 10$
$\log b = 2.8653$
$b = 733$

$$\frac{c}{\sin \gamma} = \frac{a}{\sin \alpha}$$

$a \sin \gamma = c \sin \alpha$
$\log a + \log \sin \gamma = \log c + \log \sin \alpha$
$\log a + \log \sin 65° 20' = \log 974 + \log \sin 71° 30'$
$\log a + (9.9584 - 10) = 2.9886 + (9.9770 - 10)$
$\log a + (9.9584 - 10) = 12.965 - 10$
$\log a = 3.0072$
$a = 1017$

3. $c^2 = a^2 + b^2 - 2ab \cos \gamma$

$c^2 = 95.45^2 + 67.15^2 - 2\ (95.45)\ (67.15) \cos 35° 10'$

$95.45^2 = 2 \log 95.45$

$= 2\ (1.9798)$

$= 3.9596$

$= 95.45^2 = 9112$

$67.15^2 = 2 \log 67.15$

$= 2\ (1.8271)$

$= 3.6542$

$67.15^2 = 4510$

$2\ (95.45)\ (67.15) \cos 35° 10' = \log 2 + \log 95.45 + \log 67.15 + \log \cos 35° 10'$

$= .3010 + 1.9798 + 1.8271 + (9.9125 - 10)$

$= 14.0204 - 10$

$= 4.0204$

anti log $= 10481$

$c^2 = 9112 + 4510 - 10481$

$c^2 = 3141$

$c = \sqrt{3141}$

$\log c = 1/2 \log 3141$

$\log c = 1/2\ (3.4970)$

$\log c = 1.7485$

$c = 56$

9540 9795 9595 9110

(5(9545 x)5 5(1(9596)x)10

10(5 9550 9800 9600 9120

1/2 = x/5 1/5 = x/10

2x = 5 5x = 10

x = 3 x = 2

6710 8267

10((5 6715 x)7

6720 8274

1/2 = x/7

2x = 7

x = 4

42(34(0170 1040) x)10

0204

0212 1050

17/21 = x/10

21x = 170

x = 8.1

3140 4969

10((3141)x)14

3150 4983

1/10 = x/14

10x = 14

x = 1

SOLVING TRIANGLES WITH LOGS

41 (con't)

4. B = 119° 25'
   γ = 57° 12'
   α = 3° 23'
5. c = 497.8
   α = 38° 18'
   B = 106° 20'
6. α = 19° 1'
   γ = 115° 38'
   c = 6778
7. B = 116° 16'
   a = 5987
   b = 8670
8. γ = 71° 22'
   c = 8792
   a = 5086
9. 627.6 ft.
10. a) 273.3 ft.
    b) 426.4 ft.
11. 91.82 ft.
12. 40.2 ft.

REGENTS - LOGS & EXPONENTS - pp. 42 - 44

1. 1.5754
2. 1/8
3. 5
4. 9
5. x = 1
6. x = 2
7. 1/4
8. $N = AB^3$
9. x = 27
10. x = 3
11. .2131
12. x = 2
13. (1)
14. (1)
15. (1)
16. (3)
17. (2)
18. (2)

19. $3^{2x} = 5$

    $2x \log 3 = \log 5$

    $2x\ (.4771) = .6990$

    $2x = 1.4651$

    $x = .7$

20. $\sqrt[3]{0.692}$

    $1/3 \log .692$

    $1/3\ (9.8401 - 10)$

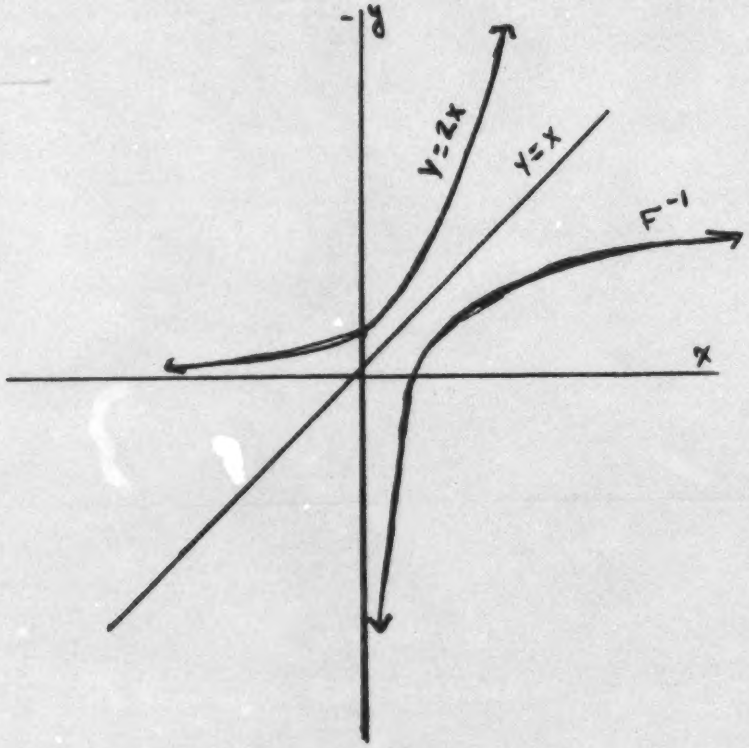
    $1/3\ (29.8401 - 30)$

    $9.9467 - 10$

    anti log = .8845 = .88

21. a & b

    c) $f^{-1} = \{(x,y)\ \ x = 2^4\}$



RTS - LOGS & EXPONENTS - PP. 45 - 46

1. 3
2. 2.6332
3. 3
4. 300
5. 8
6. 1/36
7. 64
8. $x = -1$
9. $x = 5$
10. 1
11. $x = 1$
12. (4)
13. (2)
14. (2)
15. (1)
16. (3)
17. (1)
18. (4)
19. $N = 3.14\sqrt{.432}$

$\log N = \log 3.14 + 1/2 \log .432$

$\log N = .4969 + 1/2\ (9.6355 - 10)$

$\log N = .4969 + 1/2\ (19.6355 - 20)$

$\log N = .4969 + 9.8178 - 10$

$\log N = 10.3147 - 10$

$\log N = .3147$

$N = 2.064$

$N = 2.06$

$21(^{8(}{}^{3139}_{3160}\ \ {}^{2060}_{2070})x)10$

$\frac{8}{21} = \frac{x}{10}$

$21x = 80$

$x = 4$

TS - LOGS & EXPONENTS - PP. 45 - 46

20. $N = \sqrt[3]{.875}$

$\log N = 1/3 \log .875$

$\log N = 1/3 (9.9420 - 10)$

$\log N = 1/3 (29.9420 - 30)$

$\log N = 9.9807 - 10$

$N = .9565$

$N = .96$

9805 9560

2(9807)x

4( )10

9809 9570

$1/2 = \frac{x}{10}$

$2x = 10$

$x = 5$

21. $3^x = 21$

$x \log 3 = \log 21$

$x (.4771) = 1.3222$

$x = 2.7713$

$x = 2.8$

REGENTS - LOGS & EXPONENTS PP. 47 - 49

1. 8
2. 2
3. 8
4. 1/4
5. $b^a = x$
6. 1/8
7. $x = 1/2$
8. $x = 3$
9. $\frac{4}{3}$
10. 4 1/8
11. .8840
12. 81
13. (1)
14. (1)
15. (4)
16. (4)
17. (1)
18. (4)
19. $2^x = 5$

$x \log 2 = \log 5$

$x (-3010) = .6990$

$x = 2.3223$

$x = 2.3$

20. $4^x = 12$

$x \log 4 = \log 12$

$x (.6021) = 1.0792$

$x = 1.7378$

$x = 1.7$

21. a. $\log_3 4 = x$; $3^x = 4$

$x \log 3 = \log 4$

$x (.4771) = (.6021)$

$x = 1.3016$

$x = 1.3$

b. (4)

c. (2)

RIGHT TRIANGLE TRIG
[illegible] DEFINITIONS PAGE 50

1. sine
2. cosine
3. sine
4. tangent
5. tangent
6. cosine
7. tangent
8. 8/17
9. 15/8
10. 15/17
11. 15/17
12. 8/17
13. 8/15
14. 15
15. 26
16. 19
17. 10
18. 14
19. 38
20. 23

RIGHT Δ TRIG WORD PROBLEMS
PAGE 51

1. 266'
2. 135'
3. 22'
4. 50'
5. a. 2475'
   b. 2669'
6. 2100'
7. 5800'
8. 34°
9. 37°
10. 38°

ANGLE REDUCTION/QUADS. PAGE 52

1. a. cos 23°
   b. sin 38°
   c. tan 32°
   d. csc 6°
   e. sec 13°
   f. cot 14°
   g. sin 34°
   h. cos 3°
2. a. cos 12°
   b. sin 9°
   c. tan 27°
   d. cos 41°
   e. sec 36°
   f. cot 18°
   g. sec 16°
   h. cos 23°
3. a. - cos 67°
   b. cot 15
   c. - sec 35°
   d. - tan 76°
   e. - sin 43°
   f. - csc 28°
   g. - cos 77°
   h. - cot 8°
4. a. - sec 31°
   b. tan 14°
   c. - sin 44°
   d. cos 87°
   e. cot 32°
   f. - csc 32°
   g. - cos 31°
   h. - tan 57°
5. a. II
   b. III
   c. IV
   d. II
   e. III
   f. II
   g. I
   h. III
6. a. + a. -
   b. - b. -
   c. - c. +
   d. + d. -
   e. - e. -
   f. - f. +

a. = 145 b. = 220

6. c. -, +, -, -, +, -
   d. +, -, -, +, -, -
   e. -, -, +, -, -, +
   f. +, -, -, +, -, -
   g. +, +, +, +, +, +
   h. -, -, +, -, -, +

INTERPOLATION PAGE 53

1. a. .8110
   b. .9816
   c. 1.0160
   d. 1.4644
   e. 5.4311
   f. 1.4130
2. a. .4099
   b. .1536
   c. 3.0331
   d. .5490
   e. 1.8501
   f. .9584
3. a. -.6053
   b. .0910
   c. -1.0469
   d. 6.8693
   e. -3.6955
   f. 9894
4. a. -.5700
   b. -.6540
   c. -1.0003
   d. 1.4433
   e. -1.8255
   f. .8061
5. a. 51° 24'
   b. 53° 24'
   c. 31° 45'
   d. 23° 48'
   e. 25° 29'
   f. 48° 29'
6. a. 8° 43°
   b. 2° 36'
   c. 74° 14'
   d. 29° 23'
   e. 34° 57'
   f. 24° 18'
7. a. 99° 7'
   b. 44° 11'
   c. 37° 16'
   d. 75° 18'
   e. 13° 11'
   f. 85° 53'
8. a. 24° 16'
   b. 105° 26'
   c. 28° 42'
   d. 46° 36'
   e. 69° 44'
   f. 38° 23'

DEGREE/RADIAN MEASURES
PAGE 54

1. a. $\frac{\pi}{6}$
   b. $\frac{\pi}{12}$
   c. $\frac{13\pi}{18}$
   d. $\frac{-\pi}{3}$
   e. $\frac{7\pi}{6}$
   f. $\frac{20\pi}{9}$
   g. $\frac{5\pi}{2}$
   h. $\frac{7\pi}{4}$
2. a. $\frac{\pi}{4}$
   b. $\frac{5\pi}{12}$
   c. $\frac{7\pi}{9}$
   d. $\frac{\pi}{12}$
   e. $\frac{4\pi}{3}$
   f. $\frac{13\pi}{6}$
   g. 3π
   h. $\frac{11\pi}{12}$
3. a. 45°
   b. 120°
   c. 75°
   d. 210°
   e. 114.6°
   f. -86°
   g. 540°
   h. 450°
4. a. 60°
   b. 135°
   c. 195°
   d. 150°
   e. -57.3°
   f. 28.6°
   g. 720°
   h. 630°

INVERSE FUNCTIONS PAGE 55

1. $0 = \arcsin 1/2$
2. $0 = \arccos -\frac{\sqrt{3}}{2}$
3. $0 = \arctan 2$
4. $0 = \arcsin .1$
5. $0 = \arctan \frac{-4}{5}$
6. $0 = \arccos 1$
7. $\cos 0 = 1/2$
8. $\tan 0 = -\sqrt{3}$
9. $\sin 0 = \sqrt{2}/2$
10. $\tan 0 = 1$
11. $\sin 0 = -1$
12. $\cos 0 = 0$
13. $\frac{7\pi}{6}, \frac{11\pi}{6}$
14. $\frac{7\pi}{6}, \frac{11\pi}{6}$
15. $\frac{\pi}{3}, \frac{4\pi}{3}$
16. $\pm\frac{\sqrt{3}}{2}$
17. 0
18. 1/4
19. 3/5
20. 12/13
21. .9600
22. $\sqrt{1 - a^2}$
23. $\sqrt{1 - b^2}$
24. $\frac{3\pi}{4}, \frac{7\pi}{4}$
25. $\frac{\pi}{3}, \frac{2\pi}{3}$
26. $\frac{3\pi}{4}, \frac{5\pi}{4}$
27. 0
28. $3\sqrt{3}$
29. 2/3
30. (4)
31. (3)
32. (4)
33. (2)

INVERSE FUNCTIONS PAGE 56

1. 60°, (300°)
2. 0°, (180°)
3. 45°, (225°)
4. 30°, (150°)
5. 180°
6. 150°, (210°)
7. (120°, 300°) - 60°
8. (270°) - 90°
9. 0°, (180°)
10. (225°, 315°) - 45°
11. 90°, (270°)
12. 0°
13. $\frac{\pi}{3}, \frac{(2\pi)}{(3)}$
14. $\frac{\pi}{3}, \frac{(4\pi)}{(3)}$
15. $\frac{3\pi}{4}, \frac{(3\pi)}{(4)}$
16. $\frac{\pi}{2}$
17. $\frac{(7\pi}{(6}, \frac{(11\pi)}{(6)}$ $-\frac{\pi}{6}$
18. $\frac{(3\pi}{4}, \frac{(7\pi}{4})$ $-\frac{\pi}{4}$
19. $\frac{\pi}{4}, \frac{(7\pi)}{4}$
20. $\frac{(5\pi}{6}, \frac{11\pi}{6}$ $-\frac{\pi}{6}$
21. $\frac{2\pi}{3}, \frac{(4\pi)}{3}$
22. $\frac{\pi}{3}, \frac{(5\pi)}{3}$
23. $\frac{(5\pi}{6}, \frac{11\pi)}{6}$ $-\frac{\pi}{6}$
24. $\frac{\pi}{4}, \frac{(3\pi)}{4}$
25. 0
26. 0
27. 3/5
28. $\pm\frac{12}{13}$
29. 6
30. $\frac{\sqrt{2}}{2}$
31. 1
32. $\frac{2\sqrt{2}}{3}$
33. 2
34. (2)
35. (2)
36. (3)
37. (4)
38. (3)
39. (1)
40. (4)
41. (2)

S[illegible] DIFFERENCE FORMULAS SINE:
PAGE 60

1. $\cos\theta$
2. $-\sin\theta$
3. $\frac{\sqrt{2}}{2}(\cos\theta + \sin\theta)$
4. $-\cos\theta$
5. $\frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)$
6. $-\sin\theta$
7. 1
8. 5/13
9. (1)
10. (4)
11. (2)
12. (1)
13. a. $\frac{2(1+\sqrt{10})}{9}$
    b. $\frac{2(1-\sqrt{10})}{9}$
    c. $\frac{2(\sqrt{10}-1)}{9}$
14. a. 33/65
    b. -63/65
    c. 63/65
    d. -56/65
    e. 16/65
    f. 16/65
15. 1/2
16. 1
17. $\frac{\sqrt{6}+\sqrt{2}}{4}$ (.9659)
18. $\sin 120 \cos 45 - \cos 120 \sin 45$

    $\left(+\frac{\sqrt{3}}{2}\right)\left(+\frac{\sqrt{2}}{2}\right) - (-1/2)\left(\frac{\sqrt{2}}{2}\right) = +\frac{\sqrt{6}+\sqrt{2}}{4}$
19. [illegible]in 135 cos 30 - cos 135 sin 30 =

    $\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2}\right)1/2 = \frac{\sqrt{6}+\sqrt{2}}{4}$

SUM & DIFFERENCE FORMULAS COSINE
PAGE 61

1. $\frac{\sqrt{2}-\sqrt{6}}{4}$
2. $\frac{\sqrt{2}-\sqrt{6}}{4}$
3. $\frac{\sqrt{2}-\sqrt{6}}{4}$
4. $\frac{\sqrt{2}-\sqrt{6}}{4}$
5. $\frac{\sqrt{6}-\sqrt{2}}{4}$
6. $\frac{-\sqrt{6}-\sqrt{2}}{4}$
7. $\frac{\sqrt{6}+\sqrt{2}}{4}$
8. 1
9. a. $\frac{-33}{65}$ b. $\frac{63}{65}$
10. a. $\frac{\sqrt{3}+\sqrt{15}}{8}$ b. $\frac{\sqrt{3}-\sqrt{15}}{8}$
11. $\frac{-4}{5}$
12. $\frac{-119}{169}$
13. (2)
14. (4)
15. (3)
16. (3)
17. (1)
18. (4)
19. (2)
20. (4)

SUM AND DIFFERENCE: TAN
PAGE 62

1. a. 7/11 b. 1/13
2. a. -11/8 b. 13/4
3. a. 19/30 b. 35/6
4. a. -1 b. (4)
5. 2. $+\sqrt{3}$
6. 2 $-\sqrt{3}$
7. -2 $-\sqrt{3}$
8. 2 $-\sqrt{3}$
9. (1)
10. (2)
11. (3)
12. (1)

DOUBLE ANGLES
PAGE 63

1. sin 20°
2. cos 30°
3. cos 70°
4. cos 50°
5. sin 6x
6. cos 10x
7. tan 100°
8. tan 80°
9. $\cos^2 x$
10. cos 2x
11. sin 2A = $\frac{120}{169}$

    cos 2A = $\frac{119}{169}$

    tan 2A = 120/119
12. sin 2A = $\frac{24}{25}$

    cos 2A = 7/25

    tan 2A = 24/7
13. sin 2A = 336/625
    coz 2A = 527/625
    tan 2A = 366/527
14. sin 2A = 4/5

    cos 2A = 3/5

    tan 2A = 4/3
15. sin 2A = 24/25

    sin 4A = 336/625
16. cos 2A = -7/9

    cos 4A = 17/81
17. cos 2A = -23/25
18. sin 2A = 2 15/16

HALF-ANGLE FORMULAS PAGE 64

1. $\pm\sqrt{\frac{1 - \cos x}{2}}$
2. $\pm\sqrt{\frac{1 + \cos x}{2}}$
3. $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$
4. $\frac{1 - \cos x}{2}$
5. $\frac{1 + \cos x}{2}$
6. $\frac{1 - \cos x}{1 + \cos x}$
7. 2
   3
8. 3/4
9. $\frac{1}{2}$
10. 1/10 = .1
11. I
12. I
13. II
14. II
15. (B)
16. (2)
17. (4)
18. (1)
19. (3)
20. (2)

DOUBLE & HALF ANGLES PAGE 65

1. 24/25
2. 7/25
3. 24/7
4. $\frac{3\sqrt{10}}{10}$
5. $\sqrt{10}/10$
6. 1/3
7. -24/25
8. $\pm$ 7/25
9. $\pm$ 24/7
10. $3\sqrt{10}/10$
11. $\sqrt{10}/10$
12. 3
13. cos 20°
14. cos 40°
15. tan 50°
16. sin x
17. 2 tan 2x
18. cos 8A
19. $+\sqrt{\frac{1 - \cos 20°}{2}}$
20. $+\sqrt{\frac{1 + \cos 70°}{2}}$
21. $\sqrt{\frac{1 - \cos 80°}{1 + \cos 80°}}$
22. cos 35°
23. cos 80°
24. sin 40°
25. cos x

FACTORING: PAGE 68

1. (tan x -3) (tan x + 3)
2. (sin x + 2) (sin x + 1)
3. cos x (2 cos x - 1)
4. (cot x -2) (cot x -7)
5. 2 cos x (sin x -1)
6. (tan x - 5) (tan x -3)
7. 2 sin x (cos x - 3) (cos x + 2)
8. (3 csc x - 1) (csc x - 2)
9. (5 sin x - 2) (sin x + 2)
10. (sec x + 2) (sec x - 2) ($\sec^2$ x + 4)
11. 2 (3 sin x + 2) (3 sin x - 2)
12. (scs^2 x - 1)^4 - 13 (coc^2 x - 1) + 36 =
    [(scs^2 x - 1)^2 - 9] [($\csc^2$ x - 1) - 4] =
    [(csc x - 1) -3] [(coc x - 1) + 3] [(csc x - 1) - 2
    [(cscx - 1) + 2]
    = (csc x - 4) (csc x + 2) (csc x - 3) (csc x + 1)

SOLVING TRIG EQUATIONS PAGE 69

1. = 60°
2. x = 45°
3. x = 30°
4. x = 30°, 150°, 210°, 330°
5. x = 60°
6. x = 30°
7. x = 30°
8. x = 45°
9. x = 45°
10. x = 45°, 72°
11. x = 30°, 19°
12. x = 72°
13. x = 12°
14. x = 210°, 330°
15. x = 270°, 25°
16. x = 135°, 314°, 27°, 207°

SOLVING TRIG EQUATIONS
PAGE 70

1. tan x = 3 ≈ 72°
2. sin x = .6 ≈ 37°
3. tan x = 2 ≈ 63°
4. cos x = 2/3 ≈ 48°
5. 30°, 150°, 210°, 330°
6. 37° 46'
7. x = 60°, 90°, 270°, 300°
8. x = 90°
9. x = 60°
10. x = 90°
11. x = 82°, 262°, 116°, 296°
12. x = 15°, 195°, 124°, 304°
13. x = 60°, 300°
14. x = 45°, 225°, 153°, 333°
15. x = 109°, 251°
16. x = 222°, 318°

LAW OF SINE, COSINE APPLICATIONS
PAGE 71 - 72

1. 60° 50'
2. 8.27 km
3. 4.43 km
4. 55.2
5. 51.7 km, 20 km
6. second - 60.4 km
7. 42.6'
8. 5.98 ≈ 6
9. a. 220
   b. 213
   c. 52
10. 47° 40'
11. 114° 38', 104° 44'
12. 2993.5 mi.

TABLE USE AND INTERPOLATION
PAGE 73

1. .4576
2. .9092
3. .8876
4. 3.3227
5. .7613
6. 32° 8'
7. 25° 24'
8. 10°, 170°
9. 204°, 336°
10. 9° 56', 370° 4'
11. 158° 5', 338° 5'
12. 4678

RADIAN & DEGREE MEASURES
PAGE 74

1. 2π/5
2. 126°
3. 7π/12
4. 4π/3
5. 105°
6. 13π/6
7. 240°
8. 5π/9
9. 126°
10. 3
11. 420°
12. 13π/180
13. 4π/5
14. 2π/3
15. (1)

QUADRANT PLACEMENT PAGE 75

1. (2)
2. (3)
3. (3)
4. (4)
5. (3)
6. (1)
7. (3)
8. (3)
9. (4)
10. IV
11. II
12. IV

ANGLE REDUCTION PAGE 76

1. (3)
2. (4)
3. (3)
4. (4)
5. 1
6. - sin 50°
7. - sin 10°
8. - cos 5°
9. - cos 80°
10. - cos 20°
11. tan 30°
12. - 1/2
13. - 1
14. 1/2
15. - 1

ANGLE REDUCTION PAGE 77

1. (2)
2. (1)
3. (3)
4. (4)
5. (1)
6. (3)
7. (1)
8. - sin 50°
9. - cos 50°
10. tan 25°
11. - tan 50
12. - cos 52°
13. cot 32°
14. sin 30°
15. - 1/2
16. 1/2

INVERSE FUNCTIONS PAGE 78

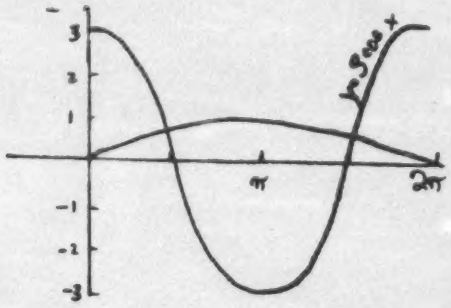
1. 1/2
2. 1
3. 2 / 2
4. 3/4
5. 5/8
6. 8/17
7. $\pi/6 = 30°$
8. $\sqrt{3}$
9. 12/5
10. 4/3
11. 1
12. $\pi/4 = 45°$
13. (3)
14. (3)
15. (3)

AMPLITUDE AND PERIOD PAGE 79

1. n
2. 2
3. 3
4. 1
5. 4n
6. (2)
7. (3)
8. (2)
9. (1)
10. (3)
11. (3)
12. (1)

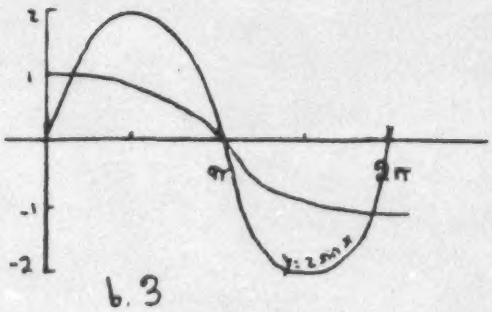
GRAPHING CIRCULAR FUNCTIONS PAGE 81

1. y = 2 sin 2x (3)
2. (4)
3. (2)
4. A 4.



b. 2

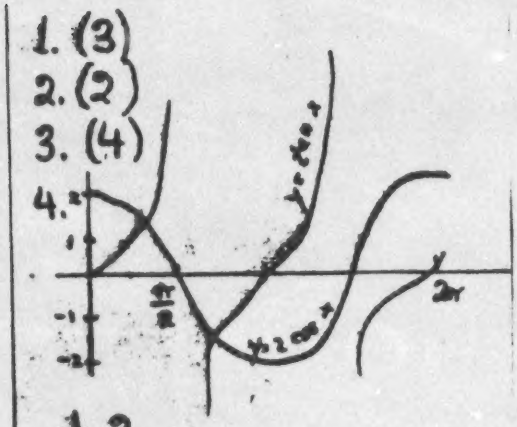
6. B 6.



b. 3

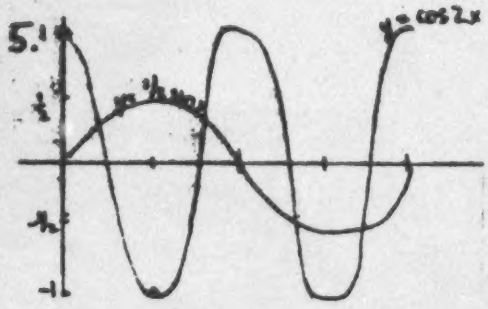
GRAPHING CIRCULAR FUNCTIONS PAGE 80

1. (3)
2. (2)
3. (4)
4.



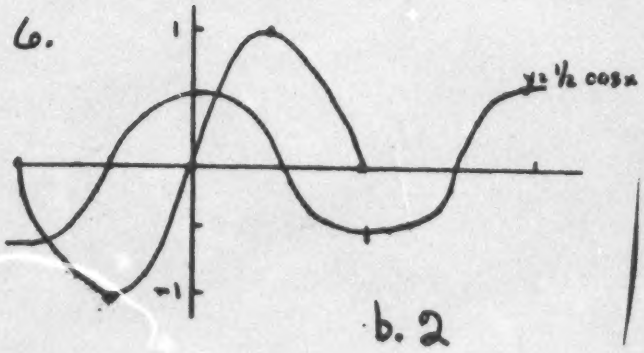
b. 2

5.



b. 4

6.



b. 2

UM & DIFFERENCE FORMULAS PAGE 83-84

1. (4)
2. (3)
3. (2)
4. (1)
5. (2)
6. -4/5
7. (3)
8. (4)
9. (2)
10. (3)
11. 3/5
12. (2)
13. (3)
14. (1)

HAVING FORMULAS PAGE 85

1. .7
2. 1/2
3. 5/6
4. $\sqrt{20}$ / 10
1. -7/18
2. 7/25
3. 120/169
4. 120/169

TRIG IDENTITY PROOFS PAGE 86

1. (4)
2. (2)
3. (1)
4. sin θ
5. -
6. -
7. -
8. -
9. -
10. a. Ω = 90°, 270°, 180°, 60°, 300°

TRIG IDENTITY PROOFS PAGE 87

1. (3)
2. (1)
3. (3)
4. (4)
5.
10. b. 90°, 210°, 330°

TRIG EQUATIONS PAGE 88

1. 0°, 180°, (360°), 30°, 150°, 210°, 330°
2. 30°
3. 90°
4. 150°
5. 180°
6. 60°
7. 30°
8. 270°
9. (4)
10. (3)
11. (3)
12. (4)

TRIG EQUATIONS - PART II QUESTIONS PAGE 89

1. 90°, 30°, 150°
2. 64°, 244°, 135°, 315°
3. 90°, 270°, 120°, 240°
4. 45°, 135°, 225°, 315°
5. 0°, 180°, 360°, 270°
6. 45°, 225°, 108°, 288°
7. 30°, 150°, 270°

TRIG EQUATIONS - PART II PAGE 90

1. x = 270°, 30°, 150°
2. θ = 270°, 30°, 150°
3. 30°, 150°, 199°, 341°
4. 180°, 31°, 329°
5. a. 2.6, .4
   b. I, III
6. 0 = 112°, 248°

LAW OF SINES PAGES 91-92

1. 10
2. 61
3. 45°
4. 9
5. 3/8
6. 9
7. 24
8. 5/6
9. (2)
10. (2)
11. 207m
12. 44
13. 70m
14. 23 cm

AMBIGUOUS CASE PAGE 93

1. 1
2. 1
3. 1
4. 2
5. none
6. none
7. 1
8. 1
9. none
10. none
11. 2
12. (2)
13. (2)
14. (3)

LAW OF COSINES - PAGE 94

1. 0
2. $\sqrt{13}$
3. 4
4. .5000
5. -.2500
6. .8750
7. -.8958
8. .7813
9. 3
10. $\sqrt{37}$

AREA - PAGE 96

1. 30 cm^2
2. 62
3. 54
4. 100
5. 25
6. a. 5.4 b. 43
7. a. 7.7 b. 12

LAW OF COSINES - PAGE 95

1. $\sqrt{19}$
2. 0
3. $2\sqrt{3}$
4. .7813
5. a. 8 b. 84°

AREA - PAGE 97

1. 31°, 270 m^2
2. 11, 58
3. 102°, 78 cm^2
4. 21°50', 99

CENTRAL & INSCRIBED ANGLES

1. a. 20 b. 40 c. 52½ d. 60 e. $\frac{a}{2}$ f. 2y
2. a. 60 b. 150 c. 45 d. 90 e. 2b f. 12a
3. 150
4. 40
5. 1:2
6. 40
7. 120
8. x = 55
9. x = 20
10. x = 30
11. x = 40, y = 90
12. x = 67½ y = 45
13. 70
14. x = 70
15. x = 50
16. x = 120
17. x = 80
18. x = 120
19. a. 115 b. 90 c. 70 d. 180 - a e. 2x
20. 84

ANGLES FORMED BY TWO CHORDS - PAGE 109

1. a. 50 b. 55 c. 77 d. 5x
2. a. 100 b. 140 c. 180 d. 75 e. 135 f. $\frac{a + b}{2}$
3. 80
4. 40
5. 100
6. x = 60
7. x = 85
8. x = 80
9. x = 140
10. x = 120
    y = 60

ANGLES FORMED BY A TANGENT AND A CHORD
PAGE 110

1. a. 20 b. 40 c. 32½ d. 70 e. b/2 f. 90 - x
2. a. 60 b. 100 c. 45 d. 140 e. 2m f. 3x
3. 40
4. 60
5. x = 30
6. x = 140
7. x = 60
   y = 120
8. x = 144
   y = 72
9. x = 220

ANGLES FORMED OUTSIDE A CIRCLE
PAGE 111

1. a. 20 b. 40 c. 60 d. 35 e. 45 f. 90
2. 150
3. 150
4. 60
5. a. 30 b. 27½ c. $\frac{a - b}{2}$ d. 2a
6. 40
7. a. 30 b. 25 c. $\frac{m - n}{2}$ d. 2x
8. 114
9. x = 20
10. x = 60
    y = 40
11. x = 80
12. x = 45
    y = 45
13. x = 90
    y = 90

ANGLE MEASUREMENT

1. a. $\overset{\frown}{AB} = 20$ b. $m\angle DAD = 40$

   $\overset{\frown}{BC} = 50$

   $\overset{\frown}{CD} = 30$

   $\overset{\frown}{DA} = 80$

2. a. 56 b. 22 c. $\overset{\frown}{BT} + \overset{\frown}{TA} = 112 + 68 = 180$

   $\overset{\frown}{BA} = 180$ $\overset{\frown}{BA}$ is a diameter

3. $\overset{\frown}{AD} = 124$ $\overset{\frown}{BC} = 144$ $m\angle BEC = 134$ $m\angle ABC = 80$

4. a.= $62\frac{1}{2}$ b =.20 c = $22\frac{1}{2}$ d = $67\frac{1}{2}$ e = 45

5. $m\angle P = 48$ $m\angle ADB = 64$ $m\angle BFC = 100$

6. $m\overset{\frown}{BD} = 24$ $m\angle BCD = 12$ $m\angle AEC = 42$ $m\angle ACB = 24$

7. a. $\frac{(4x + 2y) + (2x + 4y)}{2} = 60$ and $2x + 25 = \frac{1}{2}(2x + 4y)$

   b. $4x + 2y + 2x + 4y = 120$ and $4x + 50 = 2x + 4y$

   $6x + 6y = 120$ $2x = 4y - 50$

   $6(2y - 25) + 6y = 120$ $x = 2y - 25$

   $12y - 150 + 6y = 120$ $x = 2(15) - 25$

   $18y = 270$ $x = 30 - 25$

   $y = 15$ $x = 5$

   c. $m\overset{\frown}{AC} = 2x + 4y$ $m\overset{\frown}{BD} = 4x + 2(15)$

   $= 2(5) + 4(15)$ $= 4(5) + 2(15)$

   $= 70$ $= 50$

8. a. $3x + y = 20$ and $\frac{8x + 4y - (3x + y)}{2} = 20$

   b. $y = 20 - 3x$ $8x + 4y = 3x - y = 40$

   $y = 20 - 3(5)$ $5x + 3y = 40$

   $= 5$ $5x + 3(20 - 3x) = 40$

   $5x + 60 - 9x = 40$

   $-4x = -20$

   $x = 5$

c. $m\angle RQT = 30$

9. $(x + 15) + (8x - 40) + (6x + 10) = 360$

$15x - 15 = 360$

$15x = 375$

$x = 25$

$8x - 40 = 9(25) - 40 = 160$

$6x + 10 = 6(25) + 10 = 160$

Equal chords have equal arcs and conversely

10. $(4x - 6) + (2x + 42) + (5x - 30) + (3x + 18) = 360$

$14x + 24 = 360$

$14x = 336$

$x = 24$

$4x - 6 = 4(24) - 6 = 90$

$2x + 42 = 2(24) + 42 = 90$

$5x - 30 = 5(24) - 30 = 90$

$3x + 18 = 3(24) + 18 = 90$

LENGTH OF SEGEMENTS FORMED BY INTERSECTING CHORDS PAGE 113

1. EB = 6 2. 12½ 3. EC = 4 4. $DE = \frac{ab}{c}$ 5. CE = 8

6. ED = 2 7. 12 8. a. 8 in. b. 10 in. 9. 10 in. 10. a. $3x = 4x - 16$ $x = 16$

b. AB = 16

11. AE = 8, ED = 3, AB = 11

12. 8

13. 6

14. 6

15. 27

16. 10

17. 6

18. 12

19. 4

20. 10

21. 6

22. 5

23. a. $x(6 - x) = 9$ b. CE = 3 c. ED = 3

24. a. RE = 6 b. ES = 8 c. RS = 14

LENGTH OF SEGEMENTS FORMED BY INTERSECTING SECANTS PAGE 114

1. AD = 9 2. AE = 25 3. AE = 18 4. DE = 1 5. BC = 10 6. a. AB = 5 b. BC = 7

7. AD = 72 8. 4 9. 8

INTERSECTION OF A SECANT AND A TANGENT
PAGE 115

1. 16
2. 6
3. 4
4. 4
5. 10
6. 6
7. 2
8. 5
9. 20
10. 5
11. 6
12. $\sqrt{30}$
13. 10"
14. 9"
15. $3\sqrt{55}$
16. a. $CD = x + 6$
    b. $x(21 + x) = (x + 6)^2$
    c. $x = 4$ $BC = 4$

LENGTHS OF INTERCEPTED ARCS
PAGE 116

1. a. 3"
    b. 6"
    c. 9"
    d. 24"
    e. $4\frac{1}{2}$"
2. 20
3. a. 720° or 4π
    b. 1080° or 6π
    c. 1800° or 10π
    d. 2880° or 16π
    e. 600° or $\frac{10\pi}{3}$
4. 7
5. 26
6. 120° or $\frac{2\pi}{3}$
7. a. $\pi/4$
    b. $\pi/2$
    c. $3\pi/4$
    d. π
    e. $5\pi/4$
8. a. 9
    b. 15
    c. 18
    d. 24
    e. 36
9. 1080 or 6π

AREA OF SECTORS & SEGEMENTS

1. a. $\frac{1}{2}$
    b. 1/12
    c. 3/18
    d. 2/3
    e. 1/6
2. 6.
3. 45°
4. 10 sq. in.
5. a. 36π
    b. 18π
    c. 24π
    d. 48π
    e. 32π
6. 9π
7. a. 4
    b. 10
    c. 3
    d. 6
    e. 2
8. 9
9. a. 90°
    b. 30°
    c. 180°
    d. 40°
    e. 15°
10. a. 6"
    b. 41 sq. in.

AREA OF SECTORS & SEGEMENTS (CON'T)

11. a. 9π - 18
    b. 6π - 9 3
    c. 3π - 9
    d. 12π - 9 3
    e. $\frac{17\pi}{15}$ - 11.6

12. a. 6π - 9 3
    b. 54π - 81 3
    c. 8π/3 - 4 3
    d. 50π/3 - 25 3
    e. 25π/6 - 25 4

13. a. 6.28"
    b. 15.57"
    c. 18.84"
    d. 3.27"

14. a. 12
    b. 72
    c. 36π
    d. 36π - 72

15. 64π - 128

REGENTS REVIEW PAGE 118 - 120

1. 2
2. 6
3. 4
4. 7
5. 8
6. 70°
7. 70°
8. 3
9. 60°
10. 12
11. 10
12. 30°
13. 5
14. 2½
15. 6
16. 6
17. 9
18. 90°
19. 13.5
20. 9
21. a. 56
    b. 124
    c. 28
    d. 62.
    e. 99
22. a. 90
    b. 72
    c. 108
    d. 117

REGENTS REVIEW PART II QUESTIONS
PAGE 121 - 122

1. a. 8
   b. 2
2. a. 20
   b. 40
   c. 130
   d. 50
   e. 90
3. a. 80
   b. 40
   c. 60
   d. 80
   e. 80
4. a. 54
   b. 84
   c. 27
   d. 42
   e. 97
5. a. 80
   b. 100
   c. 70
   d. 100

REGENTS REVIEW PART II QUESTIONS
PAGE 123 - 124

1. a. 55
   b. 80
   c. 15
   d. 35
   e. 90
2. a. 50
   b. 60
   c. 20
   d. 40
   e. 90
3. a. 150
   b. 30
   c. 45
   d. 135
   e. 15
4. a. 100
   b. 44
   c. 72
   d. 20
5. a. 36
   b. 72
   c. 54
   d. 36
   e. 18
6. a. 80
   b. 40
   c. 60
   d. 80
   e. 80
7. a. 40
   b. 20
   c. 55
   d. 50
   e. 90

REFLECTIONS: COORDINATE PLANE
ANSWERS - p. 130

1. (5, -7)
2. (-1, 4)
3. (-5, 7)
4. (0, 6)
5. (7, 5)
6. (8, -3)
7. (-2, 0)
8. a. A^1 (3,0) B^1 (3, -6) C^1 (0, -6)
   b. on graph paper
   c. (2, 2) (3, 3)
9. a. A^1(1, -4) B^1 (0, 3) C^1 (-3, 4)
   b. on graph paper
   c. (0, 6)
10. a. A^1 (0, 3) B^1 (6, 3) C^1 (6, 0)
    b. on graph paper
    c. (2, 2) (3, 3)
11. a. A^1 (1, -4) B^1 (3, 0) C^1 (-3, 4)
    b. on graph paper
    c. (3,0)
12. a. A^1 (-1, 4) B^1 (-3, 0) C^1 (3, -4)
    b. on graph paper
    c. (0, 2) (0, -2)
13. a. A^1 (4, 1) B^1 (0, 3) C^1 (-4, -3)
    b. on graph paper
    c. (2,2) (-2, -2)

REFLECTIONS: COORDINATE PLANE
PAGE 131

14. x = 3
15. y = 2
16. x = -2
17. y = x
18. y = 4
19. y = x + 2
20. y = -x
21. D^1 (1,4)
    E^1 (0, 1)
    F^1 (-2, 3)
22. R^1 (0, 6)
    S^1 (1, 4)
    T^1 (4, 5)

TRANSLATIONS PAGE 132

1. (1, 16)
2. (-1, 12)
3. (-6, -3)
4. (-8, 7)
5. (3, 9)
6. (5, 1)
7. (-2, 7)
8. (0, -1)
9. (x, y) → (x + 1, y -2)
10. (x, y) → (x -1, y + 1)
11. (x, y) → (x -3, Y -4)
12. (x, y) → (x -1, y -5)
13. (x, y) → (x -7, y)
14. (x,y) → (x + 1, y -3)
15. C^1 (2, -1)
16. (2,0)
17. (x, y) (x + 1, y -3)
18. a. (5, 5) b. (0,0) c. (-1, -3)
19. b. H^1 (0, -3) O^1 (2, -3) T^1 (2, 1)
    c. H^{11} (1,1) $)^{11}$ (3, 1) T^{11} (3, 5)
    d. $T_{3,1}$
20. b. W^1 (1, 1) A^1 (5, -1) R^1 (7,3) M^1 (5, 3)
    c. W^{11} (1, 3) A^{11} (5, 1) R^{11} (7, 5) M^{11} (5, 5)
    d. $T_{5,0}$

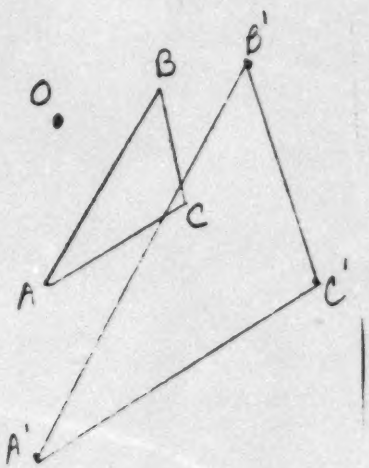
ROTATION PAGE 133

1. yes, 90°
2. yes, 60°
3. yes, 45°
4. yes, 72°
5. yes, 90°
6. yes, 90°
7. yes, 180°
8. yes, 180°
9. yes, 90°
10. yes, 90°
11. yes, 180°
12. yes, 180°
13. yes, 0 70°
14. no
15. yes, 120°
16. (3,0)
17. (-4, 1)
18. (3)

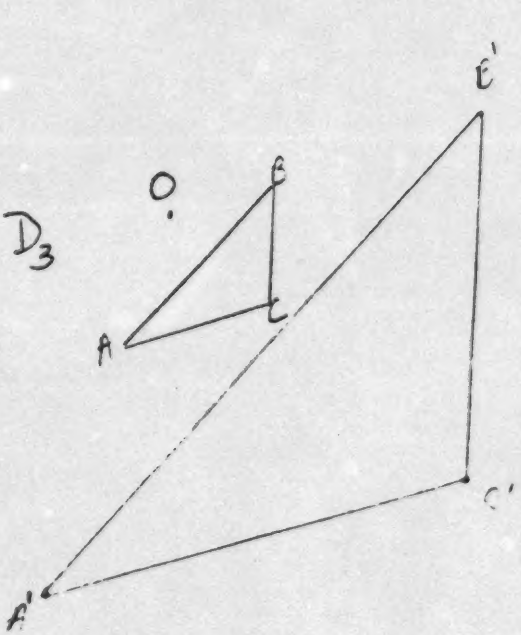
DILATIONS PAGE 134

1. S
2. P
3. OS
4. K = ½
5. 2:1

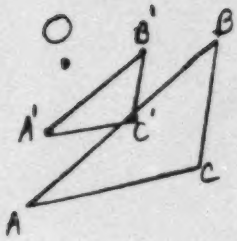
6. D_2



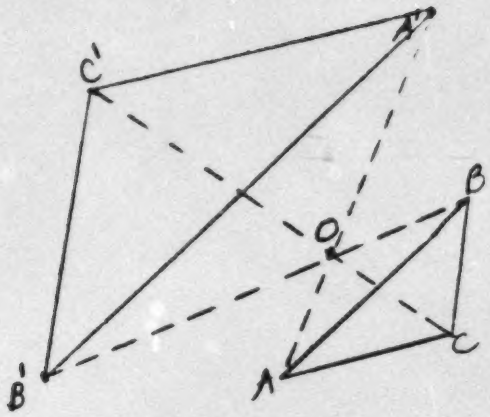
7. D_3



8. $D_{\frac{1}{2}}$



9. D_{-2}



10. a. on graph

b. A^1 (3,6) B^1 (6,6) C^1 (6,0)

c. Corresponding angles of similar triangles are congruent.

d. M (2,1) M^1 (6,3) M^D3 M^1 therefore midpoints are preserved.

e. Area $\triangle ABC = 1$ The ratio between the areas

Area $\triangle A^1B^1C^1 = 9$ is not the same as the dilation factor therefore area is <u>not</u> preserved.

11. a. on graph

b. H^1 (3, 6) O^1 (0, 0) G^1 (3,9)

c. $GH = \sqrt{0^2 + 1^2} = \sqrt{1} = 1$ $\quad GH = G^1H^1$
$G^1H^1 \sqrt{0 + 3^2} = \sqrt{9} = 3$

d. (1) $\sqrt{10}$: $3\sqrt{10}$ (2) 3:1 (3) $3\sqrt{5}$: $\sqrt{5}$ (4) $\sqrt{5}$: $2\sqrt{5}$ (5) $2\sqrt{10}$: $\sqrt{10}$

12. a. H^1 (0, 2) E^1 (3,2) N^1 (2, -1)

b. $HE = 6$ $H^1E^1 = 3$

c. 2:1

d. area $\triangle HEN = 18$

area $\triangle H^1E^1N^1 = 4\frac{1}{2}$

e. 18: $4\frac{1}{2}$

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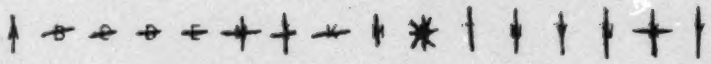
LINE SYMMETRY PAGE 135

1. 

2. b. M O M c. ~~WIKED~~ d. ~~CHECK~~ e. ~~BOB~~
   f. ~~DEED~~ h. ~~SHOWE~~ i. A A j. T O O T

3. k. l.

4. D
5. E
6. C
8. E
9. LDEC
10. $\overline{AB}$

11 - 16 See students' work

17. 4
18. 2
19. none
20. 2
21. 1
22. 1
23. infinite
24. 3

POINT SYMMETRY PAGE 136

1. a. midpoint of 1
2. c. midpoint of segment joining centers of O's
   f. midpoint of 1
   g. midpoint of vertical segment between U and N
   h. center of O
2. (2) H
3. (3) Z
4. (2)
5. (4)

SYMMETRY REVIEW PAGE 137

1. point, rotational 180°
2. none
3. line, 1
4. none
5. line, 1
6. line, 4
   point, rotational, 90°
7. point, rotational, 180°
8. line, 3, rotational, 120°
9. line, 1, point, rotational, 180°
10. rotational, 72°
11. line, 1
12. line 1
13. rotational, 72°
14. line, 1
15. line, 1
16. line, 1
17. (3) N
18. (2)
19. (2)

COMPOSITIONS PAGE 138

1. a. graph
   b. A^1 (-1, -3) B^1 (-1, -6) C^1 (-3, -3)
   c. A^{11} (1, -3) B^{11}(1, -6) C^{11} (3, -3)
   d. reflection in x - axis
2. a. A^1 (0, -2) B^1 (2, 1) C^1 (5, 4)
   b. A^{11} (0, 2) B^{11} (2, -1) C^{11} (5, -4)
   c. (3) glide reflection
3. a. C^1(0, -1) D^1(4, -1) E^1 (4, -3)
   b. C^{11} (-1, 0) D^{11} (-1, 4) E^{11} (-3, 4)
   c. (3) $R_{90°}$
4. a. A^1 (-20) B^1 (-4, 5) C^1 (-2, 6)
   b. A^{11} (2, 0) B^{11} (4, -5) C^{11} (2, -6)
   c. A^{111} (2, 0) B^{111} (4, 5) C^{111} (2, 6)
   d. (3) ry - axis
5. a. graph
   b. B^1 (-1, 1) U^1 (-1, 4) G^1 (-7, 1)
   c. B^{11} (1, -1) U^{11} (4, -1) G^{11} (1, -7)
   d. R_{-90}

COMPOSITION OF LINE REFLECTIONS PAGE 139

1. 1
2. 2
3. 3
4. 3
5. 1
6. a. A^{11} (6, 4) B^{11} (7, 2) C^{11} (9, 2)
   b. (2) $T_{6,0}$
7. a. A^{11} (-4, 4) B^{11} (-3, 2) C^{11} (-1, 2)
   b. (4) $T_{-6,0}$
8. a. A^{11} (0, -4) B^{11} (1, -6) C^{11} (3, -6)
   b. (3) $T_{0, -8}$
9. a. A^{11} (0, 12) B^{11} (1, 10) C^{11} (3, 10)
   b. $T_{0, -8}$
10. a. A^{11} (0, -4) B^{11} (-1, -2) C^{11} (-3, -2)
    b. $R_{180°}$

COMPOSITION ON LINE REFLECTIONS PAGE 139

11. a. A^{11} (6, 4) B^{11} (5, 6) C^{11} (3, 6)

    b. (1) (3, 4)

12. (2) ry $^{\circ}r_{p}$

13. (4) ry $^{\circ}r_{k}$

14. a. A^{11} (6, 4) B^{111}(7, 6) C^{11} (9, 6)

    b. (3) glide reflection

REGENTS REVIEW PAGE 141

1. a. A^{1} (-2, -1) B^{1}(-2, -4) C^{1}(-4, -2)

   b. A^{11} (6, -5) B^{11} (6, -2) C^{11} (8, -4)

   c. A^{111} (6, 2) B^{111} (6, 8) C^{111} (12, 4)

   d. F

2. a. graph

   b. A^{1}(-2, -2) B^{1} (-7, -4) C^{1} (-5, -6)

   c. A^{11} (2, 2) B^{11} (4, 7) C^{11} (6, 5)

   d. A^{111} (5, -2) B^{111} (7, 3) C^{111} (9, 1)

3. a. graph

   b. A^{1} (1, 3) B^{1} (3, 4) C^{1} (1, 6)

   c. A^{11} (-3, 1) B^{11} (-4, 3) C^{11} (-6, 1)

   d. A^{111} (3, -1) B^{111} (4, -3) C^{111} (6, -1)

4. a. graph

   b. A^{1} (-9, 1) B^{1} (-2, 5) C^{1} (-2, 1)

   c. A^{11} (1, -9) B^{11} (5, -2) C^{11} (1, -2)

   d. $R_{-90^{\circ}}$

PAGE 142

5. a. A^{1} (-2, -1) B^{1} (0, -4) C^{1} (2, -3)

   b. A^{11} (-4, 3) B^{11} (-2, 0) C^{11} (0, 1)

   c. A^{111} (-8, 6) B^{111} (-4, 0) C^{111}(0, 2)

   d. N

6. a. graph

   b. A^{1} (-2, -3) B^{1} (-9, -1) C^{1} (-5, -5)

   c. A^{11} (3, 2) B^{11} (1, 9) C^{11} (5,5)

   d. (2)

7. a. graph

   b. A^{1} (1, 4) B^{1} (4, 5)

   c. A^{11} (-4, -1) B^{11} (-1, 0)

   d. A^{111} (4, 1) B^{111} (1, 0)

   e. $T_{0, -4}$

PAGE 143

1. 2
2. 3
3. 2
4. 3
5. 4
6. 1
7. 3
8. 3
9. 3
10. 1

PAGE 144

11. D
12. (3, 5)
13. (-4, 2)
14. (2, 6)
15. C
16. (-6, 4)
17. (6, -3)
18. 1
19. BA
20. (5, 3)

REGENTS REVIEW PAGE 145

21. a. graph
    b. A^{1} (2, -1) B^{1} (6, -4) C^{1} (8, -1)
    c. A^{11} (-4, -3) B^{11} (0, -6) C^{11} (2, -3)
    d. A^{111} (-3, 4) B^{111} (-6, 0) C^{111} (-3, -2)
22. a. A^{1} (3, 3) B^{1} (9, 3) C^{1} (3, 12)
    b. A^{11} (3, -3) B^{11} (9, -3) C^{11} (3, -12)
    c. A^{111} (-3, 1) B^{111} (-3, 7) C^{111} (-12, 1)
    d. D_3

REGENTS REVIEW PAGE 146

1. 1
2. 1
3. 4
4. 2
5. 4
6. 3
7. 4
8. 1
9. 1

PAGE 147

10. (5, 1)
11. (5, 1)
12. (-6, 4)
13. (7, -1)
14. (-4, 8)
15. (3, -5)
16. (1, 2)
17. EF
18. E

PAGE 148

19. D
20. B
21. a. graph
    b. A^{1} (1, 6) B^{1} (5, 8) C^{1} (4, 5)
    c. A^{11} (-6, -1) B^{11} (-8, -5) C^{11} (-5, -4)
    d. A^{111} (9, 0) B^{111} (11, 4) C^{111} (8, 3)
22. a. A^{1} (-1, -1) B^{1}(-4, -1) C^{1} (-3, -2)
    b. A^{11} (-3, 3) B^{11} (0, 3) C^{11} (-1, 2)
    c. A^{111} (2, -2) B^{111} (8, -2) C^{111} (6, -4)
    d. W
    e. T

PAGE 149

1. 3
2. 4
3. 2
4. 1
5. 3
6. 4
7. 3
8. 3
9. 2
10. 2

PAGE 150

11. 3
12. (-2, 3)
13. (-3, -2)
14. (1, 5)
15. (-4, -2)
16. (3, -1)
17. (3, 5)
18. B

PAGE 151

19. D

20. BC

21. a. graph

b. A^{1} (0, 4) B^{1} (-1, 8) C^{1} (-4, 8)

c. (1)

d. A^{11} (4, -8) B^{11} (8, -7) C^{11}(8, -4)

e. (4)

22. a. A^{1} (9, 0) B^{1} (7, -4)

b. A^{11} (5, 8) B^{11} (1, 6)

c. A^{111} (-8, 5) B^{111} (-6, 1)

d. 1. reciprocals

2. equal

3. negative reciprocals

PERMUTATIONS PAGE 152

1. $4! = 12$
2. $10! = 3,628,800$
3. a. $26^3 = 17,576$
   b. 60
4. a. $5^3 = 27$
   b. 60
5. 30,240
6. 3,024
7. a. $9! = 362,880$
   b. $8! = 40,320$
8. $6! = 720$
9. $6 + 30 + 120 = 156$
10. $4 + 12 + 24 + 24 = 64$
11. 1,757,600
    17,576,000
    19,333,600
12. 7,436,000
13. 25,000
14. 6561
15. $2^{10} = 1024$
16. 9765625
17. a. $6! = 720$
    b. 720
18. a. 6,720
    b. 720
    c. 840

COMBINATIONS PAGE 153

1. a. 380
   b. 190
2. a. 17,160
   b. 715
3. 252
4. a. 1,860,480
   b. 15,504
5. a. 35
   b. 35
6. a. 84
   b. 84
7. 2,598,960
8. a. 336
   b. 56
9. a. 126
   b. 56
   c. 70
   d. $126 = 56 + 70 = 126$
10. 15
11. 64 (includes plain pizza plus pizzas containing one or more toppings)

CIRCULAR COMBINATIONS PAGE 154

1. $5! = 120$
2. $4! = 24$
3. $6! = 720$
4. $9! = 362880$
5. $3! \cdot 4! = 144$
6. $5! \cdot 6! = 86400$
7. $\frac{3! \cdot 4!}{7!} = \frac{144}{5040} = \frac{1}{35}$

PROBABILITIES PAGE 155

1. a. $1/4 = .25$

   b. $1/2 = .5$

   c. $3/13 = .231$

2. a. $1/2 = .5$

   b. $3/10 = .3$

   c. $4/10 = .4$

3. a. $5/36 = .139$

   b. $1/6 = .167$

   c. $5/36 = .139$

4. $2/3 = .667$

5. a. $\frac{156}{2652} = \frac{39}{676} = .058$

   b. $\frac{650}{2652} = \frac{325}{1326} = .245$

6. a. $\frac{12}{2652} = \frac{1}{221} = .005$

   b. $\frac{2}{2652} = \frac{1}{1326} = .0008$

7. $\frac{1}{52C13} = \frac{1}{\frac{52!}{13!\,39!}} = \frac{1}{261,480,000,000}$

8. $\frac{39C13}{52C13} = .013$

9. a. $\frac{3}{28} = .107$

   b. $\frac{15}{28} = .536$

   c. $\frac{10}{28} = \frac{5}{14} = .357$

10. a. $\frac{1}{28} = .036$

    b. $\frac{12}{28} = \frac{3}{7} = .429$

    c. $\frac{15}{28} = .536$

11. a. $\frac{4}{660} = \frac{1}{165} = .006$

    b. $\frac{1}{660} = .002$

    c. $\frac{6}{660} = \frac{1}{110} = .009$

12. $1/6 = .167$

13. $1/5 = .2$

14. $11/12 = .917$

15. $1/2 = .5$

AT LEAST - AT MOST PAGE 156

1. a. $\frac{4}{16} = \frac{1}{4} = .25$ b. $\frac{1}{16} = .063$

   c. $\frac{5}{16} = .313$ d. $\frac{5}{16} = .313$

2. $\frac{16}{64} = \frac{1}{4} = .25$

3. $\frac{26}{32} = \frac{13}{16} = .813$

4. a. $\frac{25}{72} = .347$ b. $\frac{125}{216} = .579$

   c. $\frac{200}{216} = .426$ d. $\frac{16}{216} = .074$

   e. $\frac{215}{216} = .995$ f. $\frac{3}{8} = .375$

   g. $\frac{1}{2} = .5$ h. $\frac{20}{27} = .741$

   i. $\frac{7}{8} = .875$ j. $\frac{18}{27} = .667$

5. $\frac{7500}{7775} = .965$

6. $\frac{171}{1296} = .132$

7. a. $\frac{512}{625} = .819$ b. $\frac{2944}{3125}$

8. a. $\frac{51}{243} = .210$ b. $\frac{192}{243} = .790$

9. $\frac{13}{125} = .104$

10. $\frac{48}{81} = .593$

11. $\frac{54}{64} = .844$

12. $\frac{11}{16} = .688$

13. a. $\frac{5}{9} = .556$ b. $\frac{4}{9} = .444$

14. a. $\frac{300}{729} = .412$ b. $\frac{304}{729} = .417$

c. $\frac{425}{729} = .583$ d. $\frac{64}{729} = .088$

e. $\frac{240}{729} = .329$ f. $\frac{665}{729} = .912$

BINOMIAL THEOREM - PAGE 157

1. $a^6 + 6a^5y + 15a^4y^2 + 20a^3y^3 + 15a^2y^4 + 6ay^5 + y^6$

   b. $a^6 - 6a^5y + 15a^4y^2 - 20a^3y^3 + 15a^2y^4 - 6ay^5 + y^6$

2. a. $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 21x^2y^5 + 7xy^6 + y^7$

   b. $x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - 4^7$

3. a. $p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

   b. $p^5 - 5p^4q + 10p^3q^2 + 5pq^4 - q^5$

   c. $32p^5 - 90p^4q + 80p^3q^2 - 40p^2q^3 + 10pq^4 - q^5$

4. a. $a^3 + a^2b + ab^2 + b^3$

   b. $x^3 - 9x^2y + 27xy^2 - 27y^3$

5. $\frac{16 + 32a + 24a^2 + 8a^3 + a^4}{16} = 1 + 2a + \frac{3a^2}{2} + \frac{a^3}{2} + \frac{a^4}{16}$

6. $32x^{15} - 80x^{12} + 80x^9 - 40x^6 + 10x^3 - 1$

7. $5^{12} + 6x^{10} + 15s^8 + 20s^6 + 15s^4 + 6s^2 + 1$

8. $a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8$

9. $20^3 + 3(20^2) + 3(20) + 1 = 9261$

10. $20^3 - 3(20^2) + 3(20) - 1 = 6859$

11. $a^{200} - 100a^{198}b + 4950a^{196}b^2 - 161700a^{194}b^3 \ldots$

12. $(3p)^{20} + 20(3p)^{19}2q + 190(3p)^{18}(2q)^2 + 1140(3p)^{17}(2q)^3 \ldots$

13. $\sin^{10}x + 10\sin^9x\sin y + \sin^8x\sin^2y + 120\sin^9x\sin^3y \ldots$

14. $\sin^{30}x - 30\sin^{29}x\cos y + 435\sin^{28}x\cos^2y - 4060\sin^{27}x\cos^3y \ldots$

15. a. 495 b. 495

    c. 1140 d. 1140

REGENTS REVIEW - BINOMIAL THEOREM - PAGE 158

1. $-120x^3$ 2. (1)
3. (2) 4. (1)
5. (1) 6. (3)
7. (1) 8. (1)
9. (4) 10. (2)
11. (3)

REGENTS REVIEW - PROBABILITIES - PAGES 159 - 160

1. (2)
2. 1/2 = .5
3. 3/4 = .75
4. 5/24 = .208
5. 1/6 = .167
6. 5/9 = .556
7. 10/64 = .156
8. $\frac{328}{625}$ = .525
9. 5/72 = .069
10. 1/9 = .111
11. 1/3 = .333
12. 13/256 = .051
13. 9/64 = .141
14. 5/32 = .156
15. 8/81 = .099

REGENTS REVIEW - PROBABILITY PART II - PAGES 161 - 162

1. a. $P_{HEADS} = 2/3$; $P_{TAILS} = 1/3$
   b. (1) 32/81 = .395; (2) 1/243 = .004; (3) 20/27 = .741
2. a. (1) 1/2 = .5; (2) 1/4 = .25
   b. (1) 1/4 = .25; (2) 54/64 = .844
3. a. (1) 3/4 = .75 ; (2) 1/4 = .25
   b. (1) 27/64 = .422; (2) 54/64 = .844
4. a. (1) 1/7 = .143 ; (2) 2/7 = .286; (3) 5/7 = .714
   b. (1) 60/343 = .175 ; (2) 275/343 = .802
5. a. (1) 1/9 = .111 ; (2) 4/9 = .444 ; (3) 5/9 = .556
   b. (1) 240/729 = .329 ; (2) 304/729 = .417

SIGMA NOTATIONS - PAGE 163

1. 42
2. 20
3. 30
4. 14
5. 36
6. 15
7. 2 1/12
8. 45
9. 55
10. 30
11. 42
12. 12
13. 1
14. 0
15. 1
16. 30
17. 120°
18. 1
19. 2
20. Answer will vary
21. a. 74
    b. 90
    c. 90
    d. 138
    e. b & c

STANDARD DEVIATION - PAGE 164
(Frequency of 1)

1. a. 24

b.

xi	$\bar{x}$	$xi - \bar{x}$	$(x_i - \bar{x})^2$
30	24	6	36
29	24	5	25
26	24	2	4
23	24	-1	1
12	24	-12	144

c. $\sqrt{42} = 6.5$

2. a. 125
   b. 4
3. a. 3
   b. $\sqrt{2} = 1.4$
4. a. 7
   b. $\sqrt{10} = 3.2$
5. a. 59
   b. $\sqrt{6} = 2.4$
6. a. 187
   b. $\sqrt{10.8} = 3.3$
7. a. neither
   b. 8.2
   c. 4.6
   d. Sean
8. a. 9.6
   b. $\sqrt{1.04} = 1$
9. a. 4
   b. 3.1
10. a. 2.5
    b. .8

STANDARD DEVIATION - GROUPED DATA
PAGE 165

1. a. 11

b.

xi	fi	fixi	$xi - \overline{X}$	$(Xi - \overline{X})^2$	$fi\ (Xi - \overline{X})^2$
15	1	15	4	16	16
12	4	48	1	1	4
10	2	20	-1	1	2
9	3	27	-2	4	12

c. $\sqrt{1.8} = 3.4$

2. a. 24

b. $\sqrt{9.4} = 3.1$

3. a. 28

b. $\sqrt{76} = 8.7$

4. a. 7

b. $\sqrt{4.4} = 2.1$

5. a. 67

b. $\sqrt{12} = 3.5$

6. $\sqrt{10.75} = 3.3$

7. a. 30

b. $\sqrt{6.2} = 2.5$

c. yes

8. a. 37

b. $\sqrt{16.8} = 4.1$

c. 3.6

9. a. 83

b. $\sqrt{89.2} = 9.4$

10. a. 81

b. $\sqrt{82.54} = 9.1$

NORMAL DISTRIBUTION PAGE 167

1. 50th
2. 68%
3. 16th
4. 95%
5. a. 94
   b. 88
   c. 76
   d. 70
6. 4
7. 4
8. 4
9. 2
10. a. 2
    b. 1
    c. 2
11. 3
12. a. none
    b. Garbowski
    c. Novak
    d. Backer
    e. none
    f. Oestrich
13. a. 1.7
    b. 92
14. a. 7
    b. 11.5
15. a. 2.4
    b. 11.6
16. a. 8
    b. 61

REVIEW PAGE 169

1. 32
2. 14
3. 45
4. 0
5. 4
6. 25
7. a. 31
   b. 32
   c. 32
8. 2
9. 3
10. a. Len 8/9
    Stan 4/9
    b. Len
11. 4
12. 3
13. a. 12.5
    b. 67.5
    c. 92.5
14. a. 5
    b. $\sqrt{6.8} \approx 2.6$
15. $\sqrt{18.8} \approx 4.3$
16. a. 45
    b. 50
    c. 50
    d. $\sqrt{60} \approx 7.7$
17. a. 91
    b. $\sqrt{44} \approx 6.6$

REGENTS REVIEW SIGMA NOTATION PAGE 171

1. 32
2. 5
3. 20
4. 18
5. 30
6. 9
7. 36
8. 1
9. 33
10. 42
11. 26
12. 50

m_i	f_i	$m_i f_i$	$\bar{x}$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
80	1	80	91	-11	121	121
85	2	170	91	-6	36	72
90	4	360	91	-1	1	4
95	0	0	91	9	81	243
100	3	300				

REGENTS REVIEW - STANDARD DEVIATION
PAGE 172

1. a. 65
   b. 68
   c. 69
   d. $\sqrt{26.2} \approx 5.1$
2. a. 29
   b. 30
   c. $\sqrt{65} \approx 8.1$
3. $\sqrt{66} \approx 8.1$
4. $\sqrt{3.8} \approx 1.9$
5. $\sqrt{51.8} \approx 7.2$
6. $\sqrt{29} \approx 5.4$
7. $\sqrt{18} \approx 4.2$
8. $\sqrt{22.4} \approx 4.7$
9. $\sqrt{24} \approx 4.9$
10. $\sqrt{1.84} \approx 1.4$
11. $\sqrt{17.4} \approx 4.2$
12. $\sqrt{2.45} \approx 1.6$
13. $\sqrt{15.2} \approx 3.9$

REGENTS REVIEW STATISTICS PAGE 174

1. 580
2. 53
3. 2
4. 95%
5. 4
6. 3
7. 1
8. 57
9. 85
10. 0
11. 68%
12. 21
13. 3
14. 4
15. 68%
16. 4
17. 4
18. 95%
19. 58
20. 2
21. a. 75
    b. $\sqrt{95} \approx 9.7$
22. $\sqrt{26.8} \approx 5.2$